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Simulation of Ionospheric Effects from Acoustic Waves Produced by Explosive Events at Ground Surface

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Introduction: acoustic- and gravity-wave response of the atmosphere

- **Explosive events at the ground (earthquakes, volcano eruptions, man-made explosions) excite the whole spectrum of atmospheric waves**
- **Acoustic- and Gravity- waves are responsible for propagating the energy to considerable distances**

Dispersion relationships for Acoustic/Gravity-wave modes of atmospheric waves

$$\omega^2 = \frac{c_s^2(k^2+k_a^2)}{2}\left[1 \pm \sqrt{1-\frac{4\omega_B^2(k^2-k_z^2)}{c_s^2(k^2+k_a^2)^2}}\right] = W_{A/G}(\mathbf{k})$$

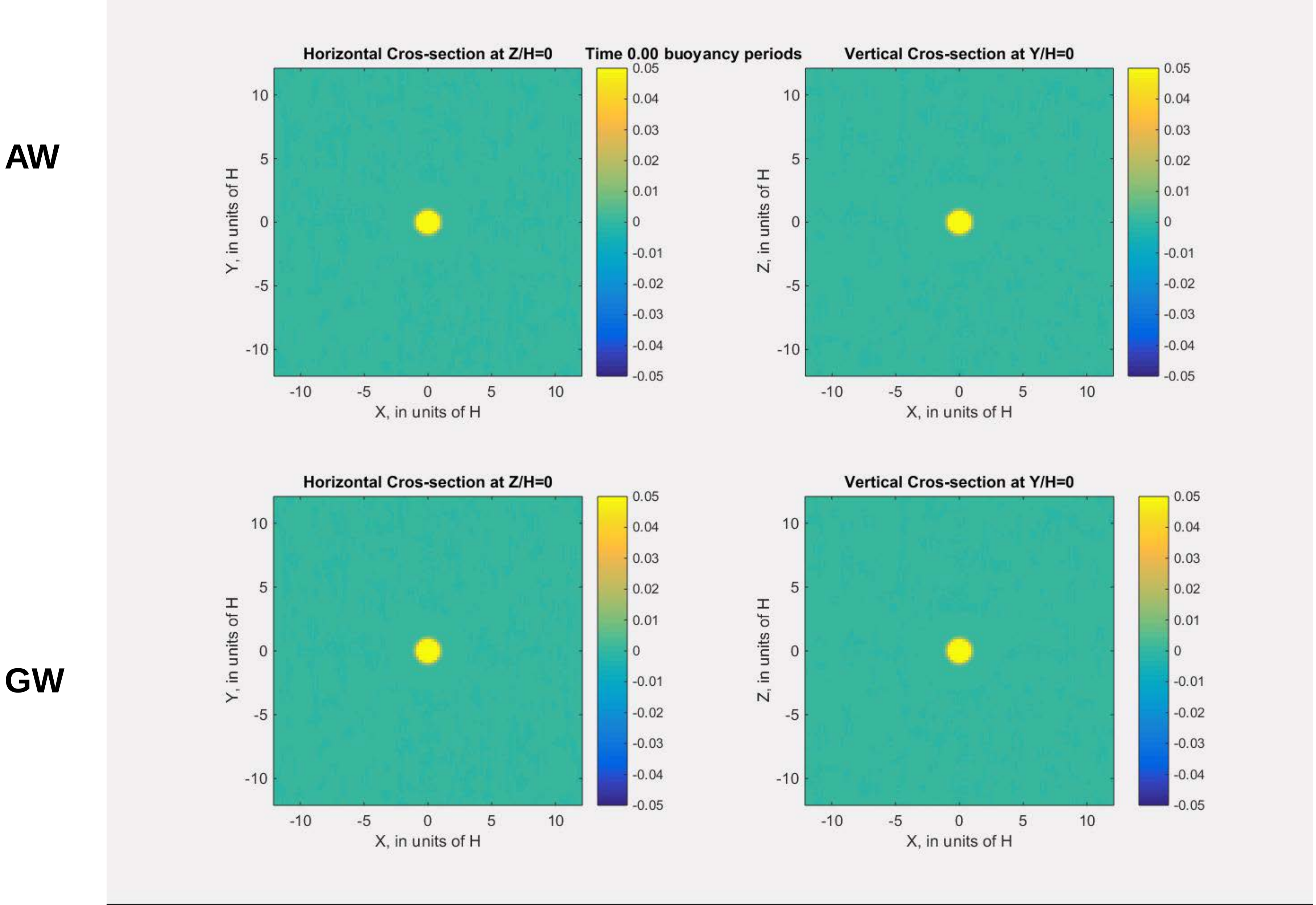
$$H = c_s^2/\gamma g \quad k_a = \gamma g/2c_s^2 = 1/2H \quad \omega_B = \sqrt{\gamma - 1}\, g/c_s$$

- **Compare propagation of medium-scale acoustic- and gravity- wave modes initiated by localized disturbances**
  - **Given an initial perturbation $P(\mathbf{r}) = e^{-r^2/(H/2)^2}$ determine evolution of the perturbation at t>0, assuming that the perturbation excites acoustic or gravity waves only**

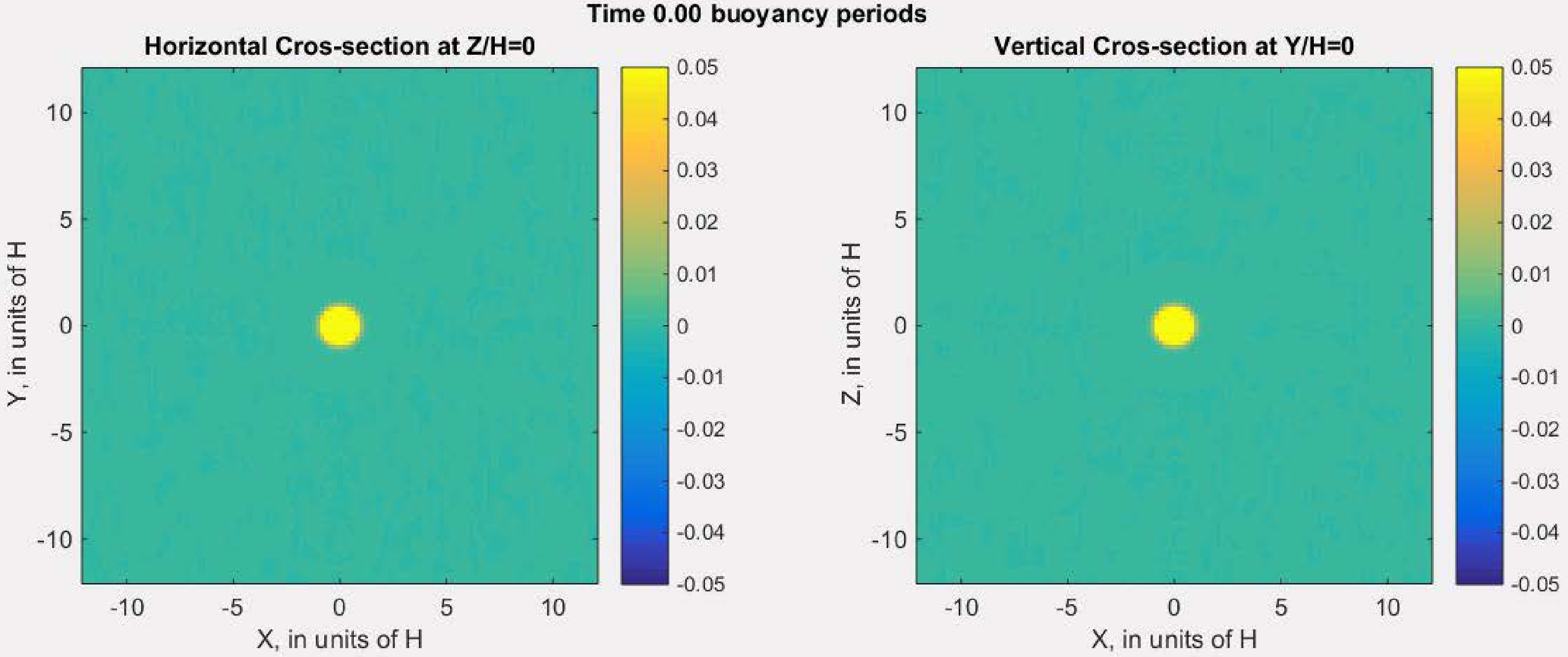
$$P_{\mathbf{k}} = \iiint P(\mathbf{r}) e^{-i\mathbf{k}\cdot\mathbf{r}} d^3\mathbf{r}$$

$$p(\mathbf{r},t) = \frac{1}{(2\pi)^3}\iiint e^{i\mathbf{k}\cdot\mathbf{r}} P_{\mathbf{k}} \cos\left(t\sqrt{W_{A/G}(\mathbf{k})}\right) d^3\mathbf{k}$$

Evolution of AW and GW Modes



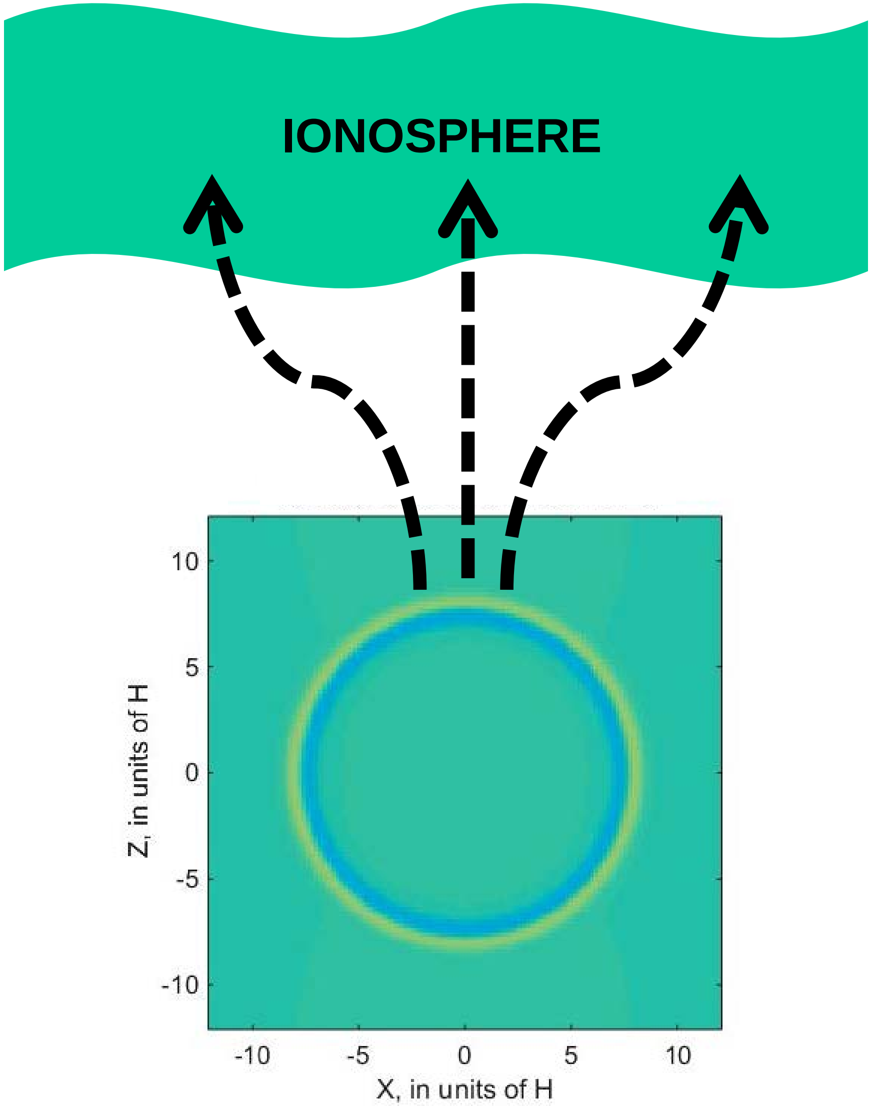
Further Evolution of the GW Mode (not a subject of this paper)



The subject of this paper-
propagation and ionospheric impact of the AW

- **Physics-based propagation model of the acoustic (infrasonic) pulse up to altitudes typical for the ionospheric F-region**
  - **Employ realistic model of the atmosphere**
  - **Account for dissipation of the pulse due to viscosity and thermal conductivity**

- **Model for ionospheric impact of the acoustic pulse**



Infrasound Propagation Model - Ray Optics

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- **The geometrical optics approximation has been widely used for modelling infrasound waves (i.e. *Godin* 2014) as well as gravity waves (*Vadas* 2007)**
- **Our implementation of the geometrical optics solution is tailored for simulating the spatial-temporal fields produced by impulsive localized sources**

The temporal-spatial acoustic field is assembled out of time-harmonic components specified by the complex amplitude $F(\mathbf{r},\omega)$ and the phase $\varphi(\mathbf{r},\omega)$:

$$f(\mathbf{r},t) = \mathrm{Re}\int_0^\infty F(\mathbf{r},\omega) e^{i[\varphi(\mathbf{r},\omega)-\omega t]} d\omega$$

$$F(\mathbf{r},\omega) = A(\theta,\phi,\omega) G(\mathbf{r},\omega) e^{-\mu(\mathbf{r},\omega)}$$

- $G(\mathbf{r},\omega)$ – focusing factor
- $A(\theta,\phi,\omega)$ - radiation pattern of the source
- $\mu(\mathbf{r},\omega)$ - attenuation index

$$\frac{d}{d\tau}\mathbf{R} = -\frac{\partial \mathrm{H}}{\partial \mathbf{k}} \Big/ \frac{\partial \mathrm{H}}{\partial \omega}$$

$$\frac{d}{d\tau}\mathbf{k} = \frac{\partial \mathrm{H}}{\partial \mathbf{R}} \Big/ \frac{\partial \mathrm{H}}{\partial \omega}$$

$$\frac{d\varphi}{d\tau} = \mathbf{k}\frac{d}{d\tau}\mathbf{R}$$

$$\frac{d\mu}{d\tau} = \chi$$

$$\mathrm{H} = \frac{c_s^2(k^2+k_a^2)}{4}\left[1+\sqrt{1-\frac{4\omega_B^2(k^2-k_z^2)}{c_s^2(k^2+k_a^2)^2}}\right] - \frac{\omega^2}{2}$$

is the Hamiltonian

τ is the group delay

The **attenuation rate** $\chi(\mathbf{R},\mathbf{k},\omega)$ is expressed in terms of viscosity and thermal conductivity coefficients following the dissipative dispersion relationship presented by (*Godin* 2014)

The **focusing factor** G is expressed using the ray tube power flow concept (*Nickisch* 1988)

Construction of the spatial-temporal solution

The above ray tracing equations are solved for a dense set of exit direction (θ, ϕ) and frequency values ω. Thus we obtain $\mathbf{R}(\tau, \theta, \phi, \omega)$, $\varphi(\tau, \theta, \phi, \omega)$, $\mu(\tau, \theta, \phi, \omega)$, $G(\tau, \theta, \phi, \omega)$

The vector equation $\mathbf{R}(\tau, \theta, \phi, \omega) = \mathbf{r}$ is resolved with respect to (τ, θ, ϕ).

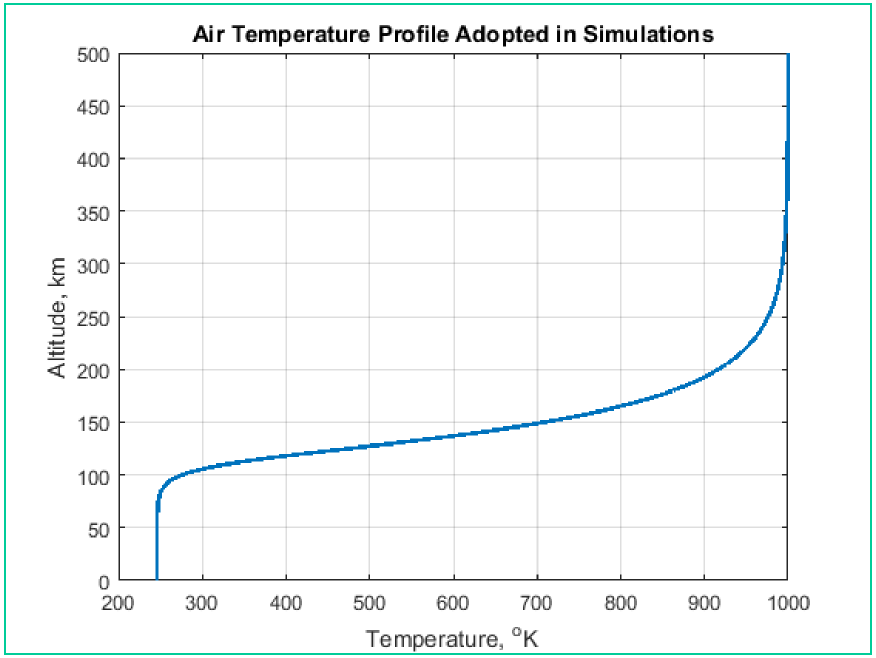
As a result we derive $\tau(\mathbf{r}, \omega)$, $\theta(\mathbf{r}, \omega)$, $\phi(\mathbf{r}, \omega)$. These functions allow to express all remaining components of the ray tracing solution as functions of $\mathbf{r}$ and ω: $\varphi(\mathbf{r}, \omega)$, $\mu(\mathbf{r}, \omega)$, $G(\mathbf{r}, \omega)$

Finally, the spatial-temporal behavior for each component of the hydrodynamic field is determined using the inverse Fourier transform

$$f(\mathbf{r}, t) = \operatorname{Re} \int_0^\infty F(\mathbf{r}, \omega) e^{i[\varphi(\mathbf{r}, \omega) - \omega t]} d\omega$$

Simulation of the infrasonic field

- Assume horizontally stratified unperturbed atmosphere
- The atmosphere is defined by the following temperature profile (*Vadas and Fritts* 2006)



- The ray tracing equations solved at $\omega = \frac{2\pi}{3600}[1,2,\ldots,3600]$ so that the interval between consecutive frequency samples is1 hr^{-1}, and the maximum frequency is1 Hz (minimum period 1 s, maximum period 1 hr)
- At each frequency the rays traced from the source at 251 elevation angles from 9.5^o to 90^o (average step of 0.3^o)
- Ray solutions are interpolated over 500 km x500 km spatial grid with 2x2 km spacing
- Be aware of fundamental and numerical limitations

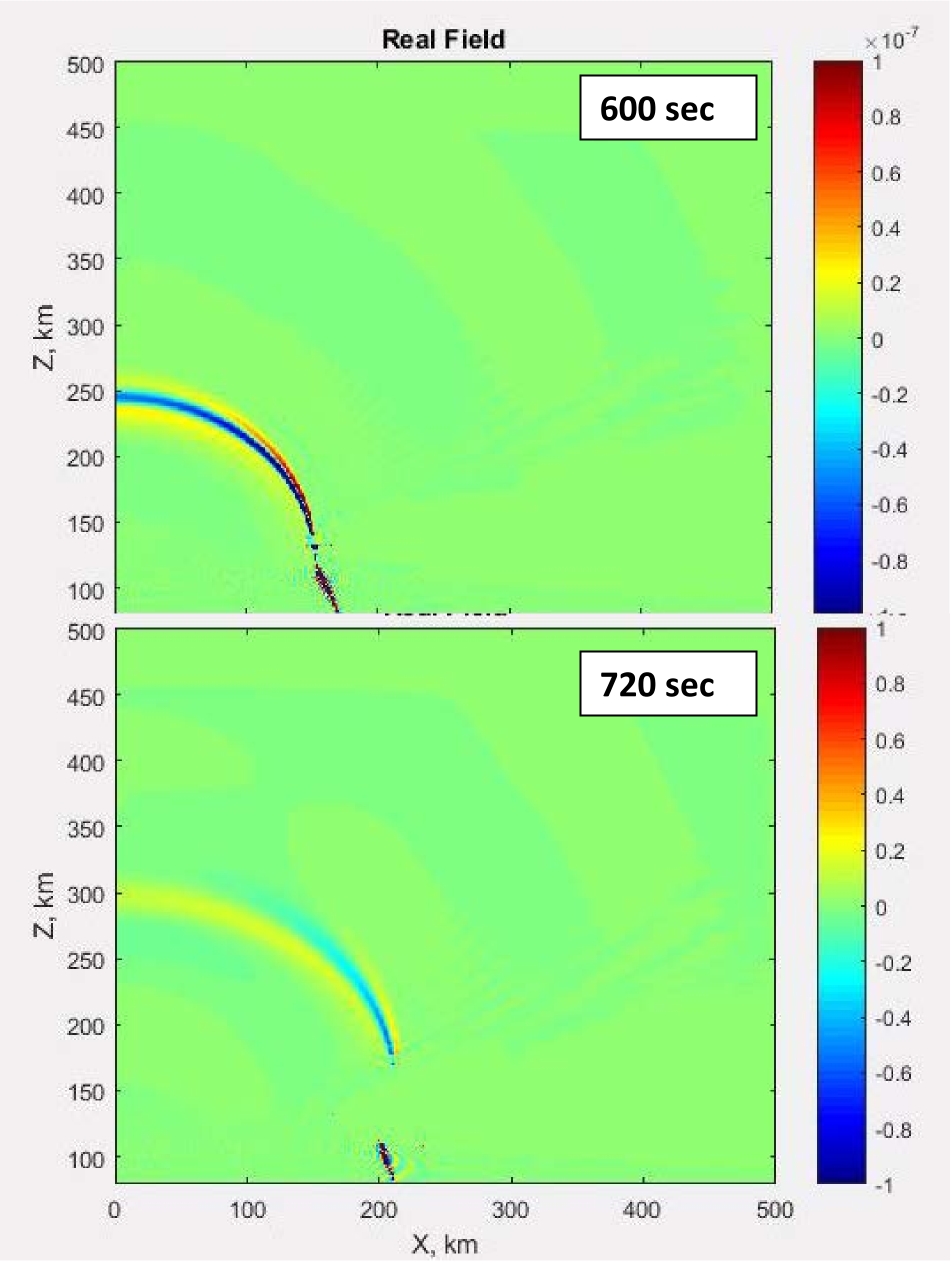
Simulation of the infrasonic field

Impact at t=0, x=0, y=0, z=0

Impact strength is 1 kg of air compressed into the origin of the coordinate system (~10 kg TNT~)

Evolution of the solution for relative perturbation of atmospheric density.

Time after the impact is indicated in each panel.



Ionospheric manifestation of atmospheric infrasound

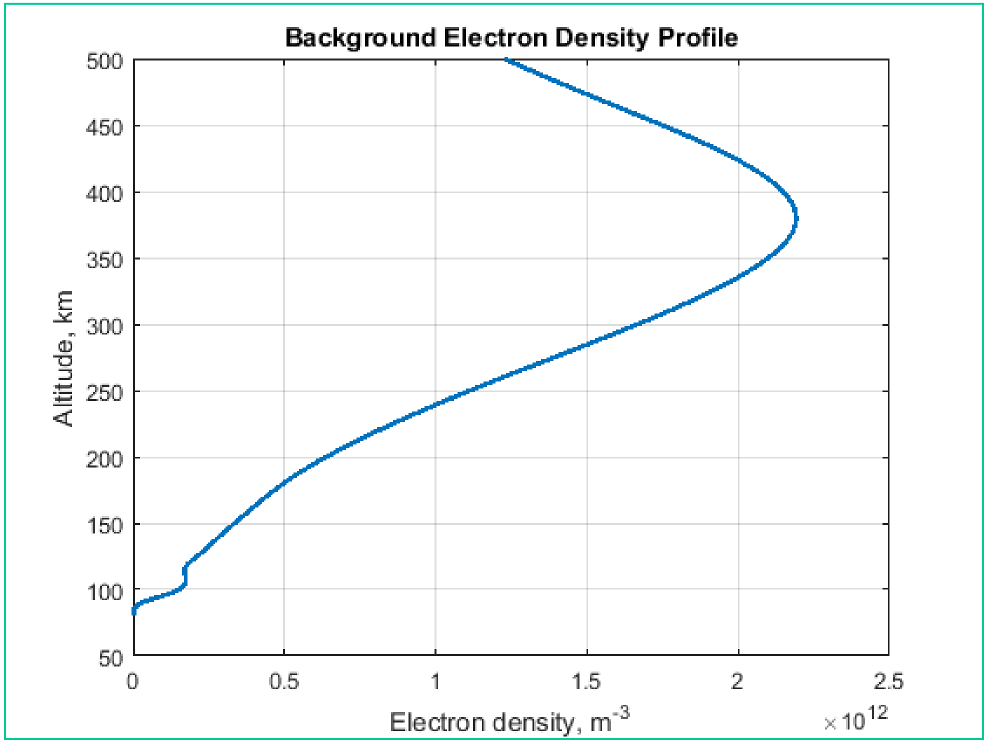
- When infrasonic waves travel through the ionosphere, the electrons and ions are dragged by the motion of neutral air. The direction of motion of charged particles is confined to the direction along the geomagnetic field. Following (*Yeh and Liu* 1972)

$$\frac{\partial}{\partial t} n_e + \nabla \cdot \hat{\mathbf{b}}(\hat{\mathbf{b}} \cdot \mathbf{v}) n_e = 0$$

- Employ linearized approximation and assume horizontally-stratified unperturbed ionosphere, then

$$\delta n_e(\mathbf{r}, t) = -N_e \hat{\mathbf{b}} \cdot \int_{-\infty}^{t} dt' \nabla(\hat{\mathbf{b}} \cdot \mathbf{v}) - \frac{\partial N_e}{\partial z} \hat{b}_z \hat{\mathbf{b}} \cdot \int_{-\infty}^{t} dt' \mathbf{v}$$

- The electron density profile $N_e(z)$ employed in the simulation (represents daytime conditions according to IRI2007)



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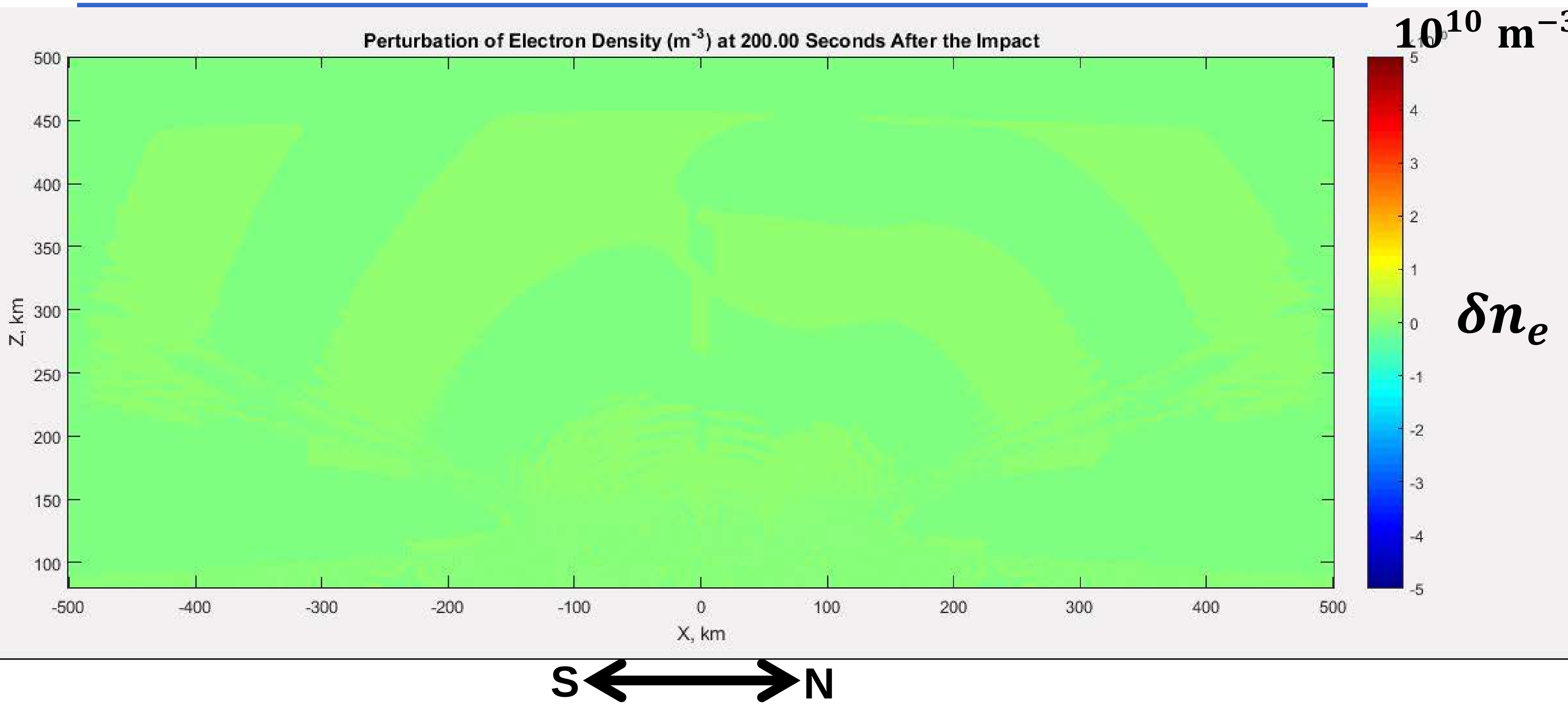
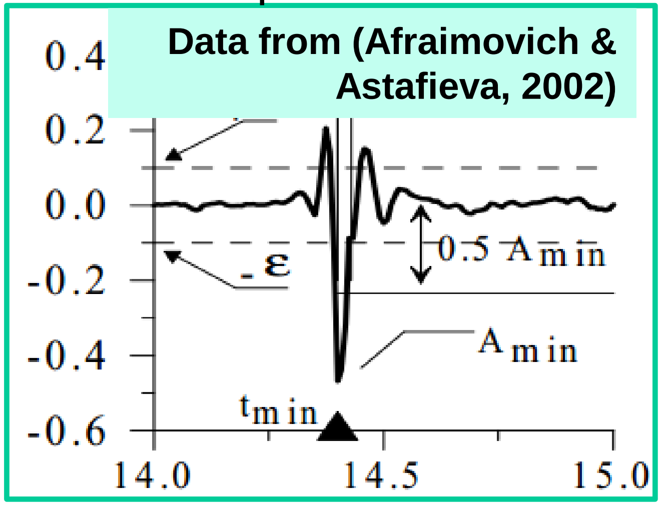


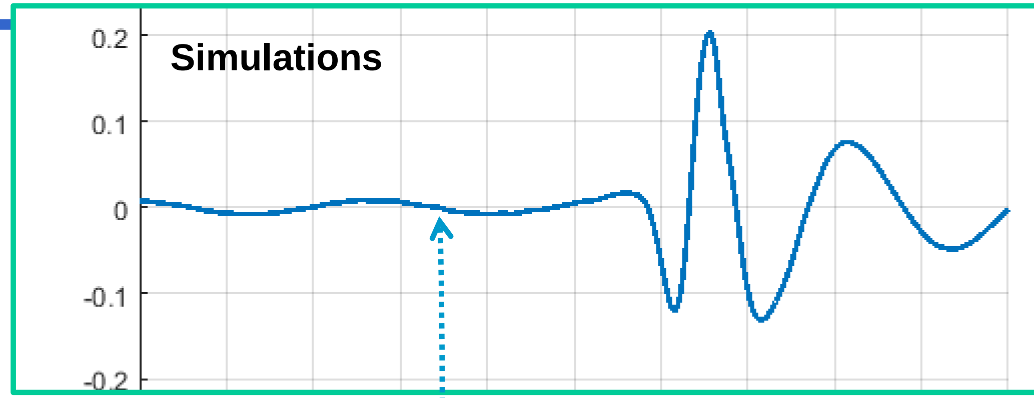
Vertical cross-sections of electron density perturbations along the plane of magnetic meridian for the atmospheric perturbation shown earlier.

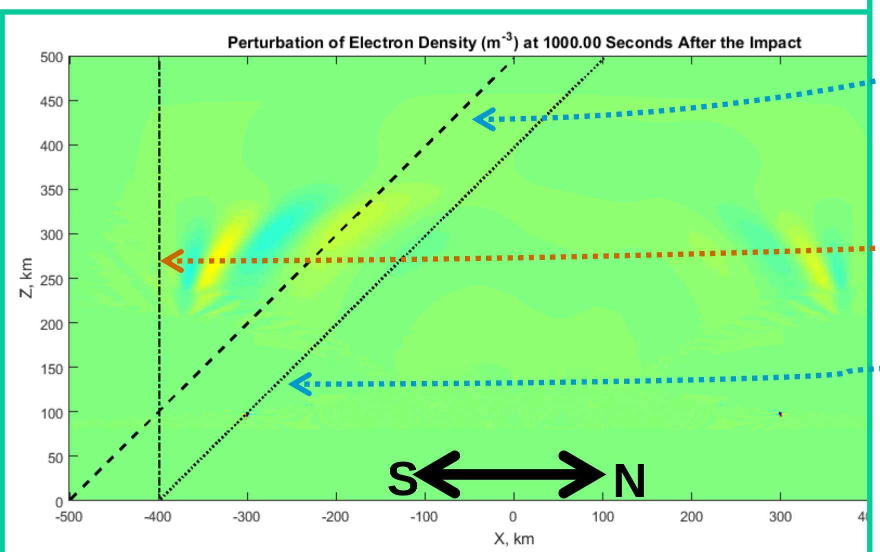
The horizontal axis shows ground-distance from the impact location
The strength of the impact is scaled up to $3x10^6$ kg of air (~30 kt ~)

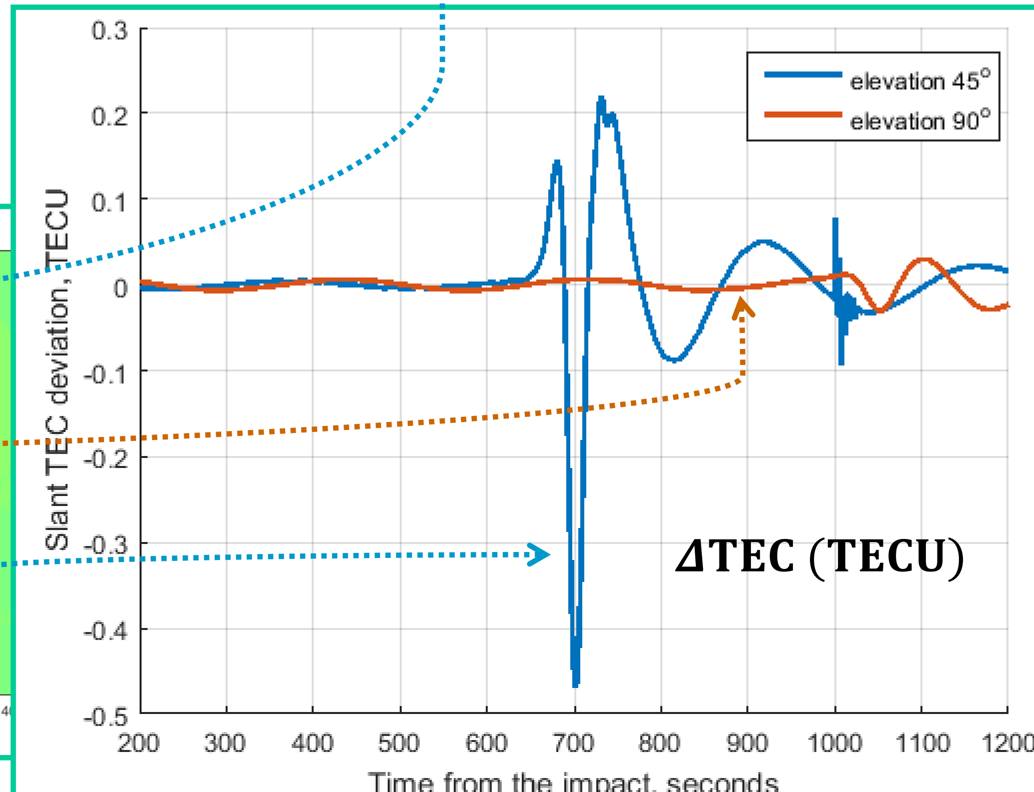
Manifestation in time series of TEC deviations

"isolated ionospheric disturbances"









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Conclusions

- **Created a model for propagation of infrasonic pulse radiated by explosive events.**
- **Simulated ionospheric effects of the infrasonic pulse.**
- **Simulated effects of the infrasound radiated by earthquakes on TEC measurements appear to be consistent with observations.**