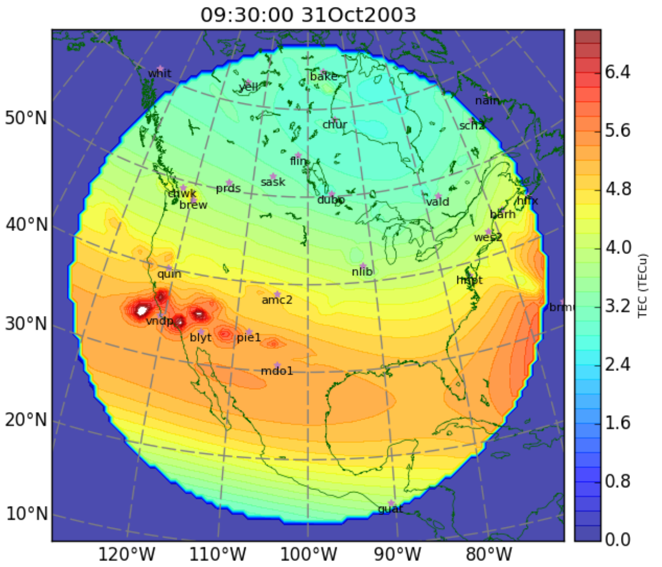
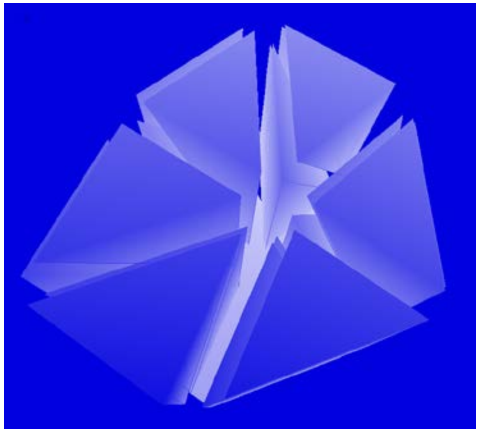
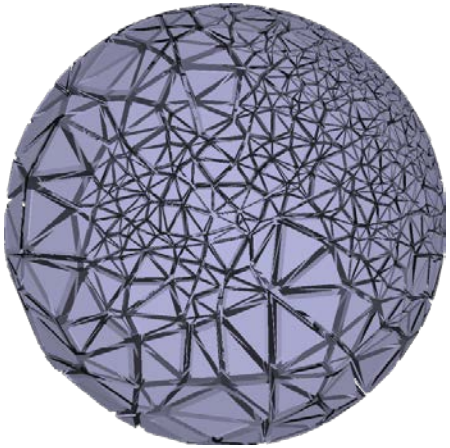
3D Ionospheric Tomography using Finite Elements







Dr Richard Penney &
Dr Natasha Jackson-Booth

May 2017

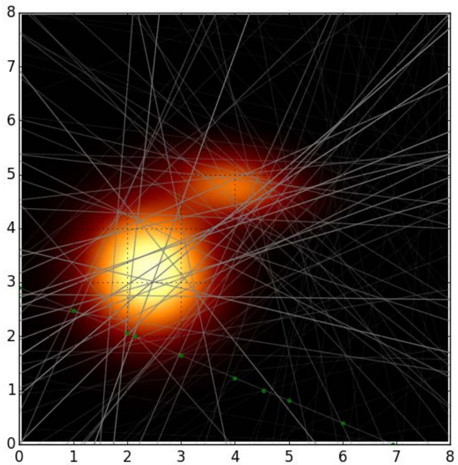
Agenda

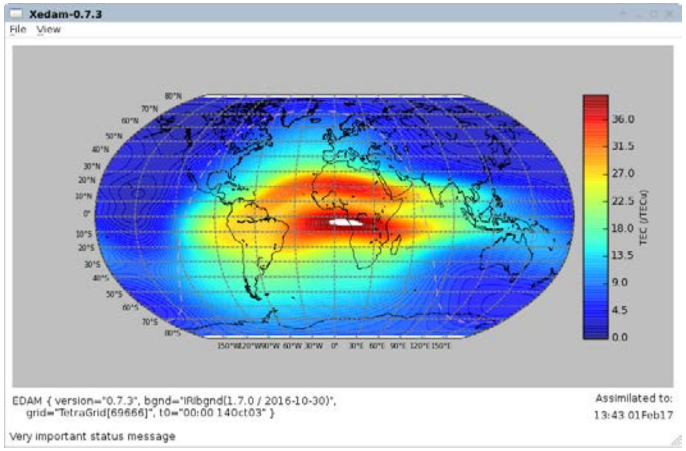
- **Goals of ionospheric imaging**

- **Tomography on 2D model**
  – Interpolation artefacts
  – Self-consistent tomography

- **3D finite-element tomography**
  – Tetrahedral grids
  – Pure GPS least-squares fits
  – Mixed IRI+GPS assimilative model

- **Outlook**





Ionospheric imaging

- **3D ionospheric density profiles would be extremely powerful in understanding ionospheric effects**
  - e.g. RF ray-tracing, HF reflection heights, GPS time-offsets, etc.

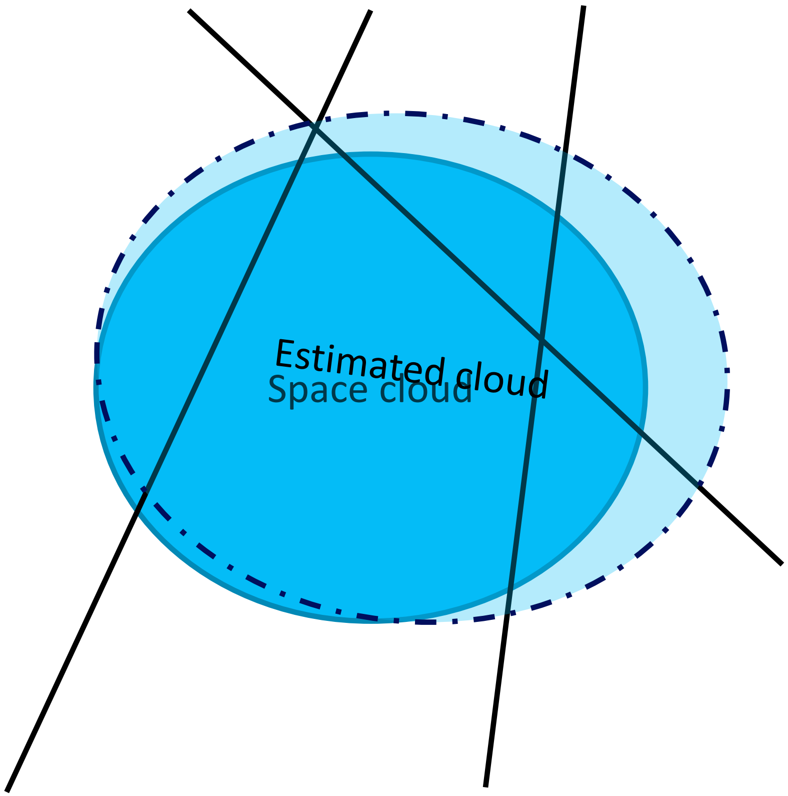
- **Direct measurement of 3D electron densities is not practicable**
  - Assessing estimated densities against "truth" is also challenging

- **Density profile must be inferred by some form of model-fitting process from sparse measurements**
  - Dual-band GPS, ionosonde traces, etc.

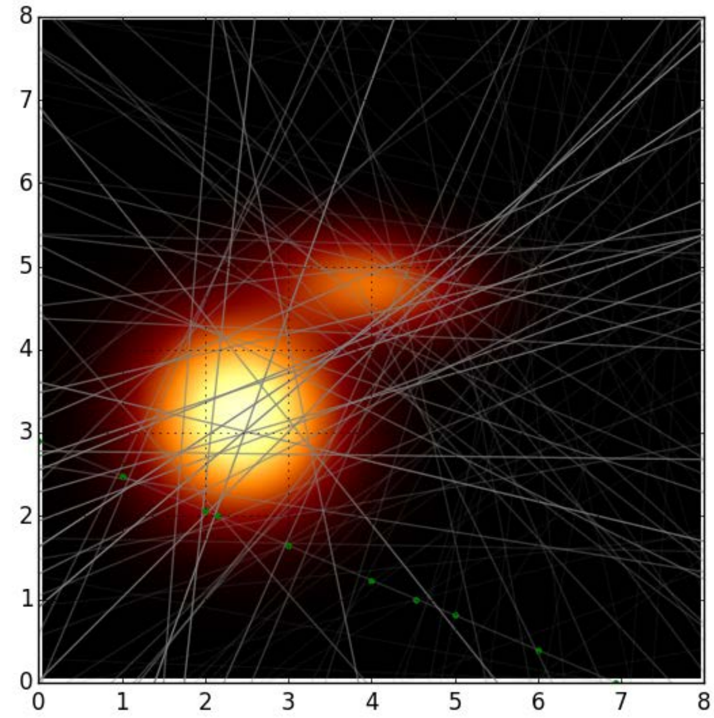
- **Many tomographic models have been developed using various approaches**
  - e.g. empirical orthonormal functions, 2D vertical slices, 2½D shells, iterative or exact least-squares, assimilative Kalman filters, etc.

- **Optimal use of limited sensor data requires careful control of artefacts created by the fitting process**

Idealized 2D assimilation of GPS TEC



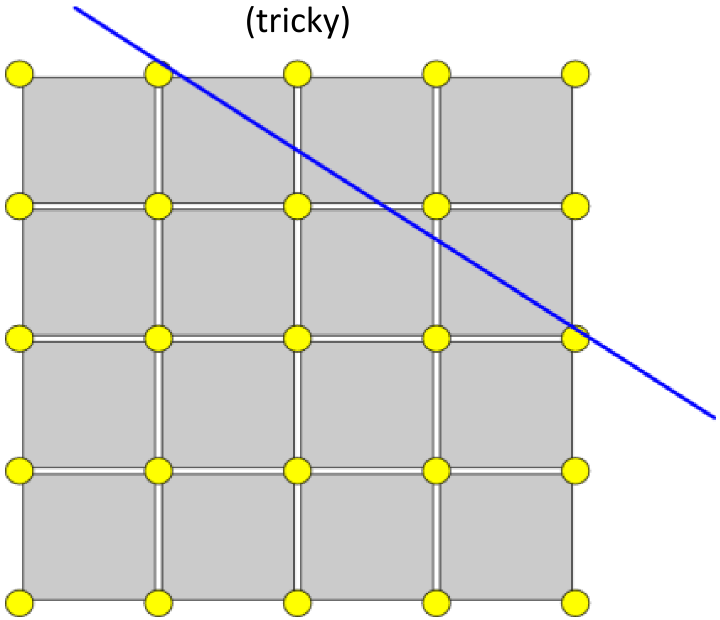
TEC measurements
(line integrals of density)



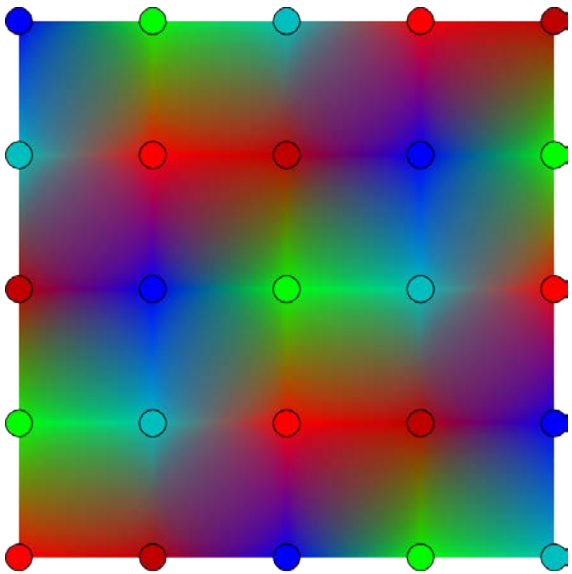
Simulated 2D scenario
2 clouds, 100 rays, 9x9 grid

Two distinct interpolation phases

Fitting
(tricky)



Rendering
(easy)



- **Each observation needs to be linked to the grid-cells through which it passes**

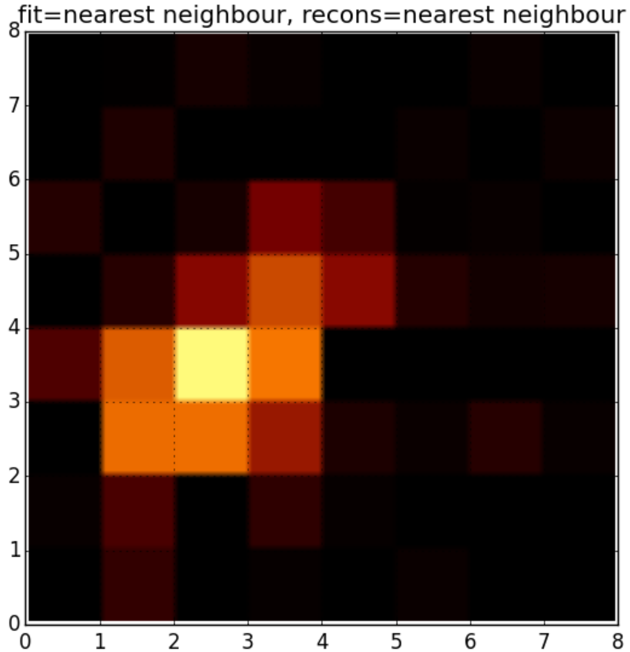
- **The node-weights need to be adjusted to best-fit the measurements**
  - This requires interpolating between nodes to compare with the measurements

- **A rendered image can be constructed from the fitted corner weights**

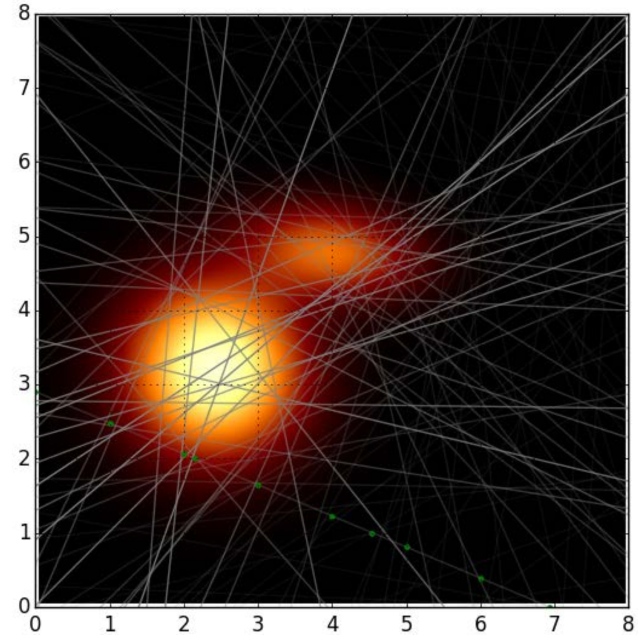
- **This may have many more pixels than there are nodes in the grid**
  - This typically involves regular interpolation within grid cells

Tomographic fit (naïve interpolator)

Tomographic estimate



Truth

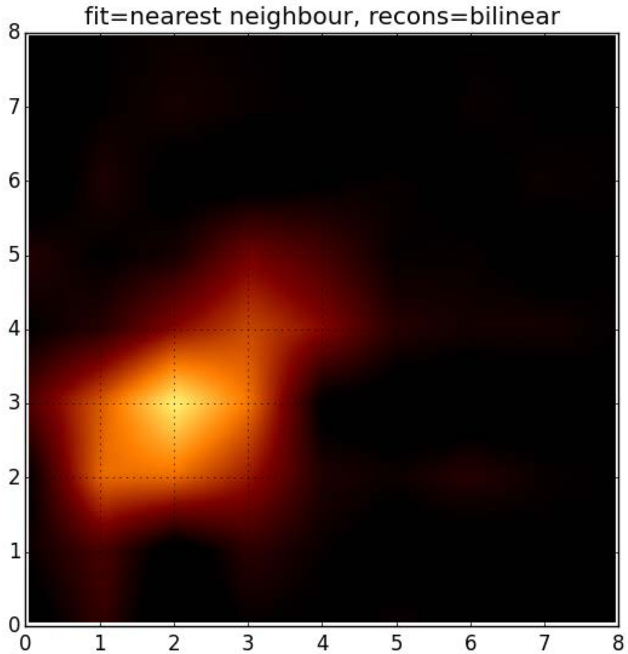


- **Piecewise-constant nearest-neighbour interpolator produces very crude image**
  - Strong discontinuities
  - Ambiguities over shape & number of clouds
  - Strong blurring
  - Spurious cloud-like features

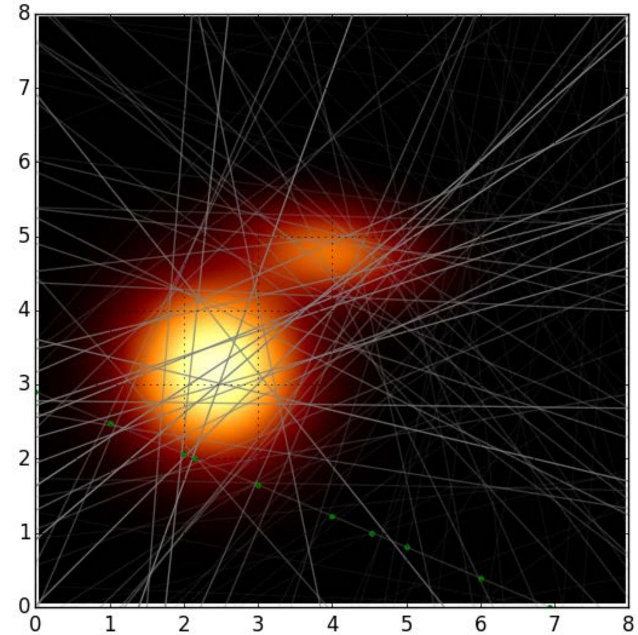
Tomographic fit (naïve post-processing)

QinetiQ

Tomographic estimate



Truth

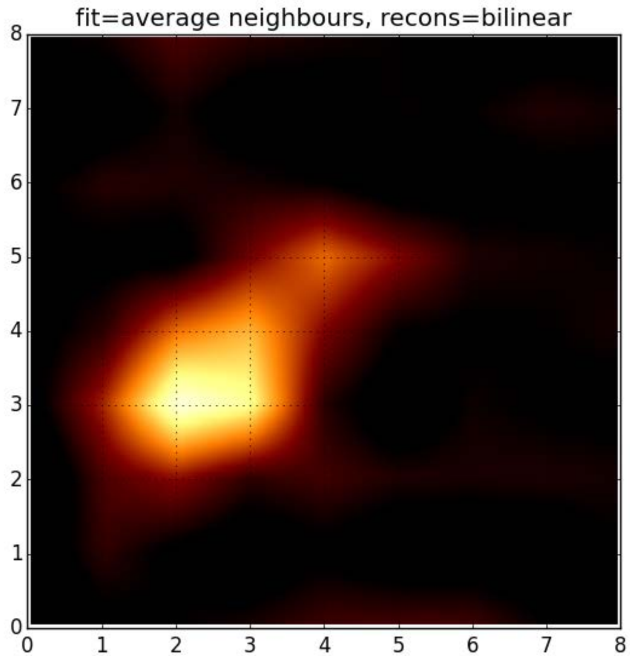


- **Tomographic fit, and final image, can use different interpolation schemes**
  - Here, piecewise-constant for fitting, bi-linear for rendering

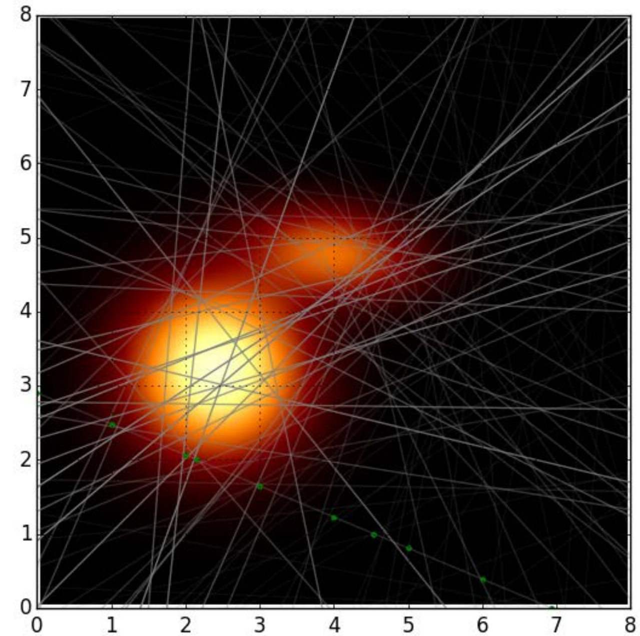
- **Still poor resolution of cloud features, blurring, misplacement of cloud centre, etc.**

Tomographic fit (non self-consistent)

Tomographic estimate



Truth

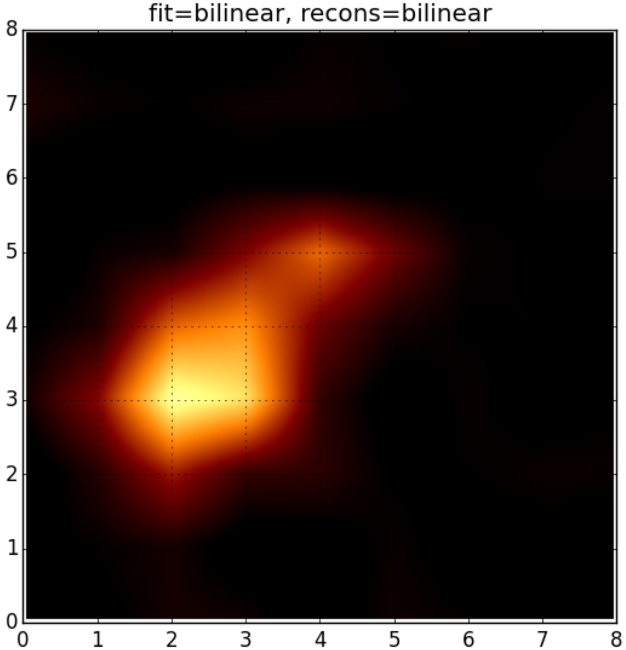


- **Using mismatched fitting/rendering interpolators gives choice of different artefacts**
  - Here, corner-average for fitting, bi-linear for rendering

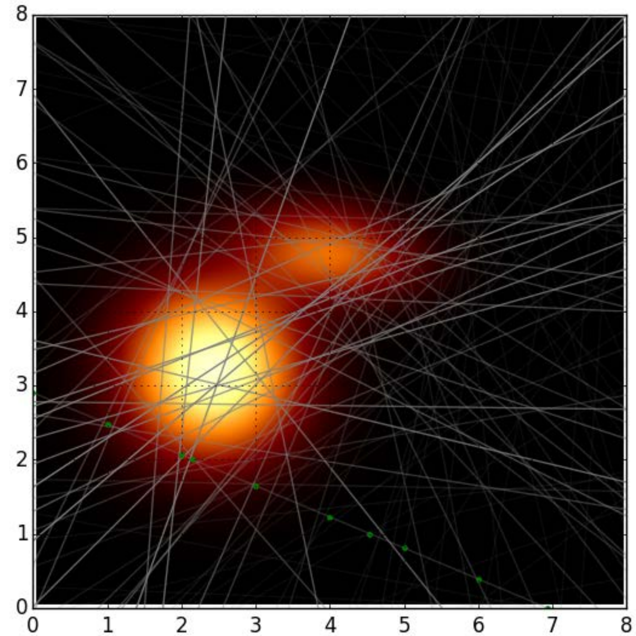
- **Still poor resolution of cloud features, blurring, etc.**
  - Cloud centre happens to be more accurate

Tomographic fit (smoothly self-consistent)

Tomographic estimate



Truth



- **Using the same interpolation function in fitting & rendering produces the highest quality images**
  - Requires more complex algebra, but computational cost is almost identical
  - Makes optimal use of limited sensor data

Tetrahedral grids for 3D tomography

- **Optimal tomographic fitting requires being able predict how each interpolation weight would affect any GPS TEC measurement**
  – This requires being able to efficiently integrate each basis-function along a ray between satellite and receiver

- **Choosing suitable 3D basis-functions (or grids) is challenging**
  – Undesirable choices include: piece-wise constant; latitude/longitude/altitude grids; etc.

- **Tetrahedral grids with piecewise-planar interpolation have attractive theoretical properties:**
  – Efficient calculation of line-integrals for GPS TEC assimilation
  – Naturally avoid unphysical discontinuities in electron density
  – Allow multi-resolution grids, e.g. to give finer coverage of operational regions
  – Require no special handling of polar regions (unlike rectangular latitude/longitude grids)

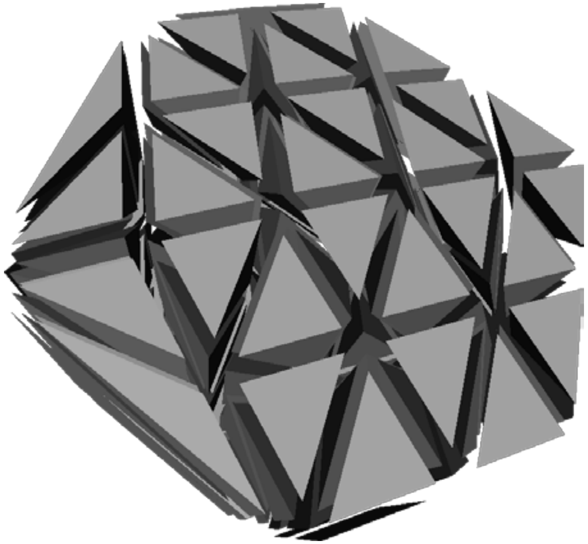
Tetrahedral grid manipulation

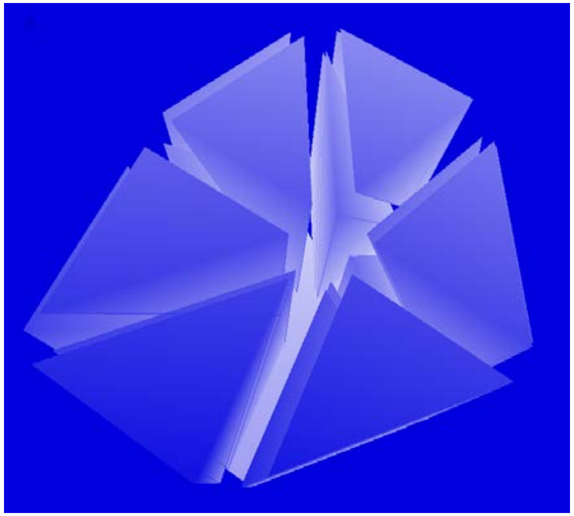
- **3D tetrahedral grids need to avoid gaps or overlaps between adjacent tetrahedra**

- **Grids can be generated by subdividing cuboids**
  - Not ideal for conforming to Earth's curvature

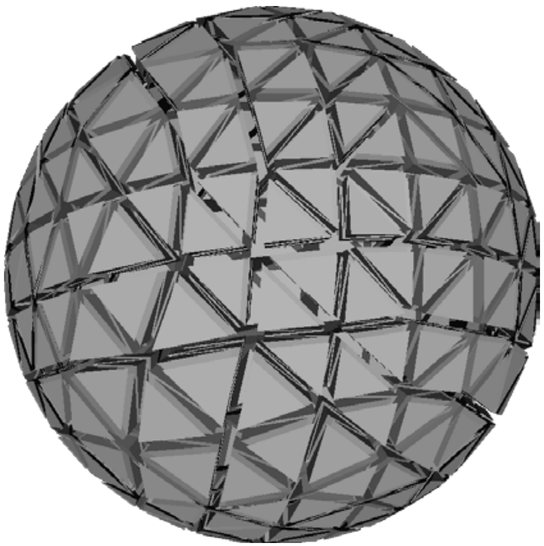
- **More flexible approach is to fit tetrahedra to a set of sample points**
  - Generate sheets of sample points at fixed altitudes
  - Use Delaunay triangularization to find optimal set of tetrahedra filling the convex hull

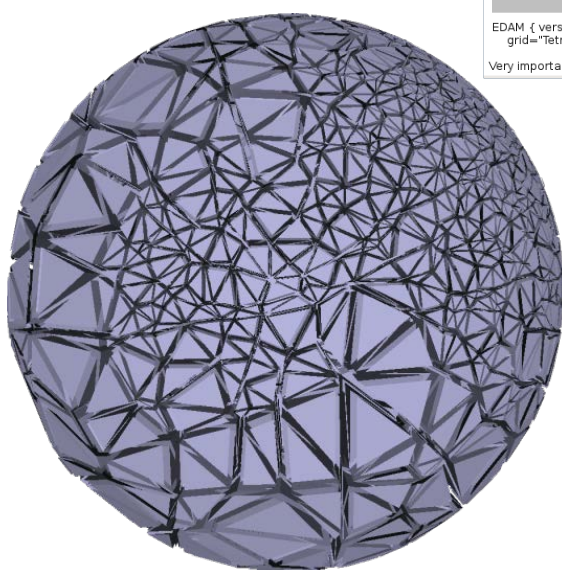
- **Specialized indexing is necessary to allow efficient density queries**

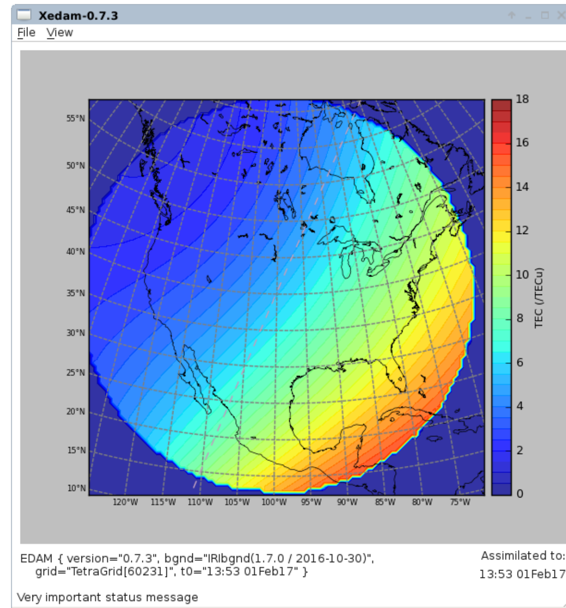


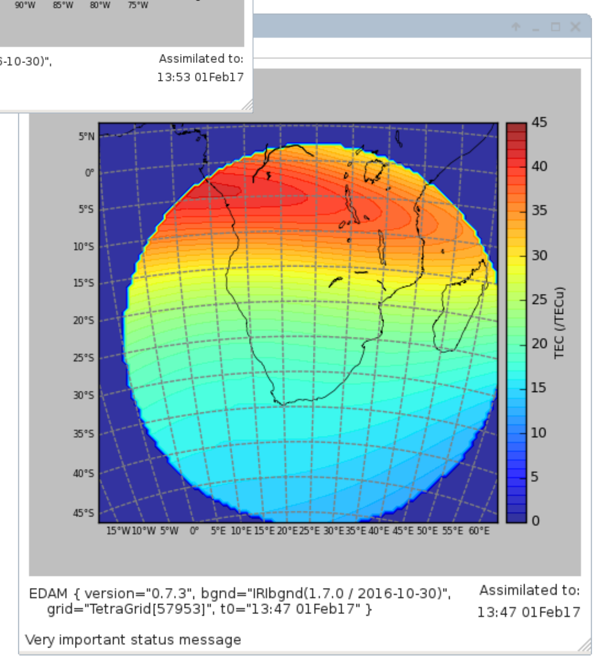


Example global & local tetrahedral grids



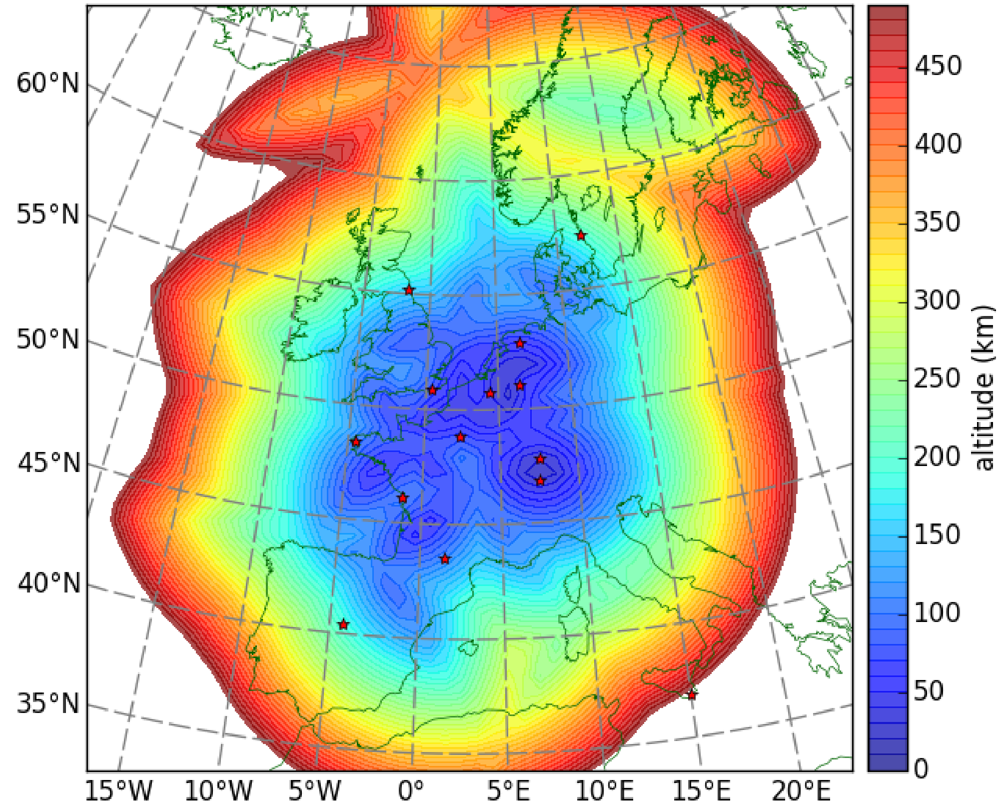




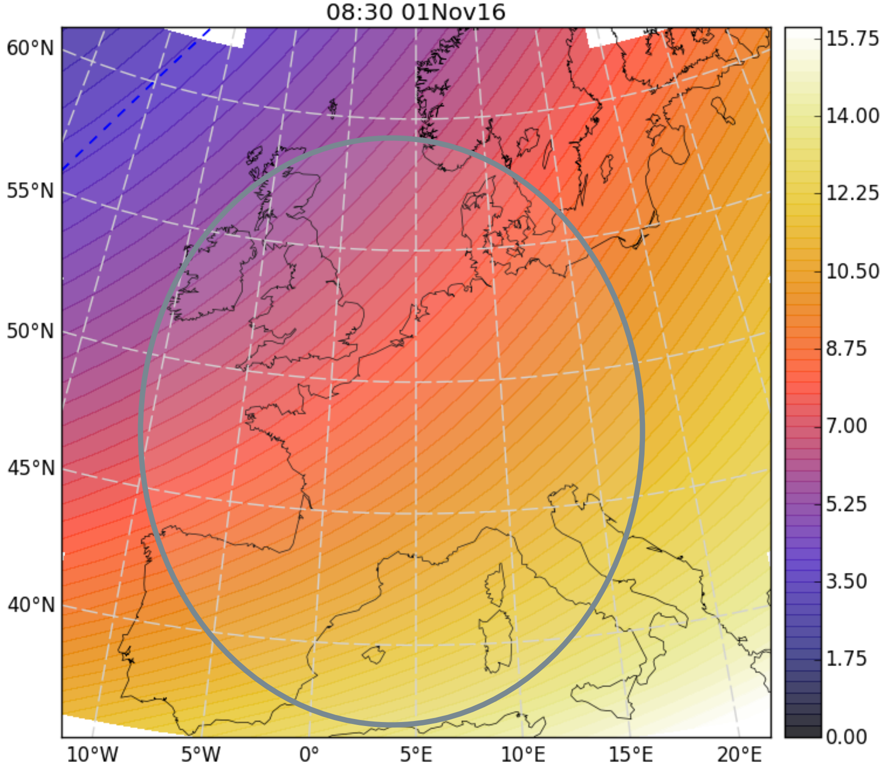


Bottom-side visibility using GPS TEC

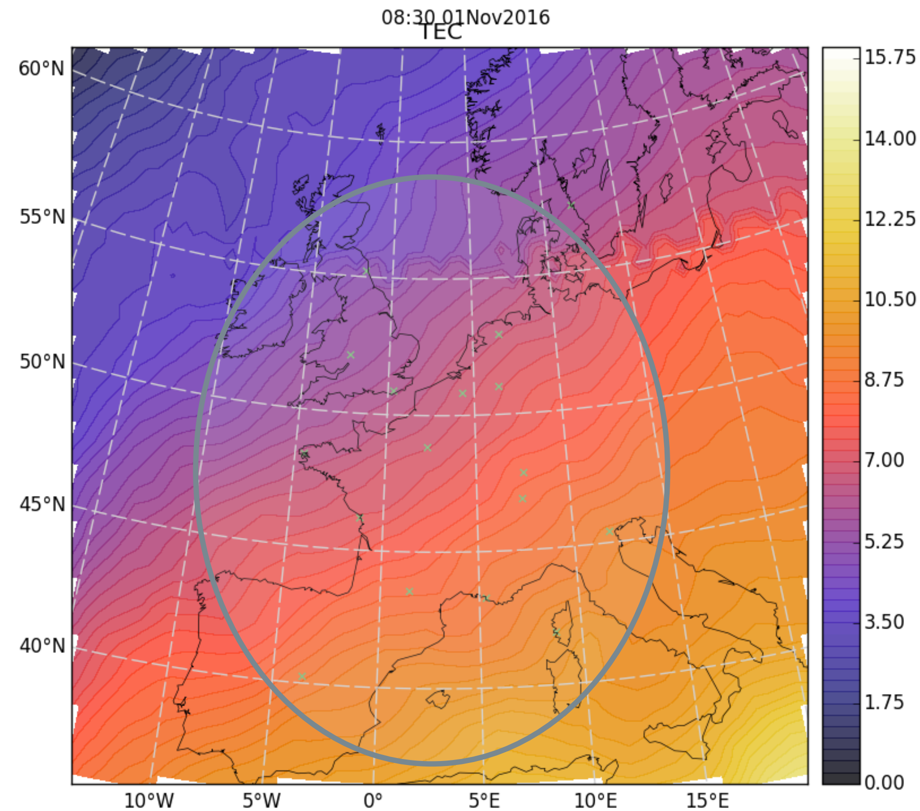
- **GPS tomography relies on having multiple look-directions through any point of interest**
- **At low altitudes, only a single GPS receiver may have visibility due to curvature of the Earth**
- **Simulated scenario has been generated using IRI-2016 and realistic GPS orbits**
  - Provides access to complete information about "correct" tomographic fit
  - Provides clean GPS time-series without need to estimate biases



Tomographic fits to synthetic IRI dataset



“True” TEC from IRI-2016



TEC from tomographic fit to synthetic GPS data

Finite-element assimilation via Bayesian inference

- **Historical data provides valuable guide to 3D electron-density profile**
  - Especially useful to “fill-in” gaps in GPS coverage
  - Useful as soft constraint on typical shape of density profiles

- **Assimilative model of ionospheric electron density can use IRI as a background (statistical prior)**

- **Finite-element model can be used as multiplicative correction to background**

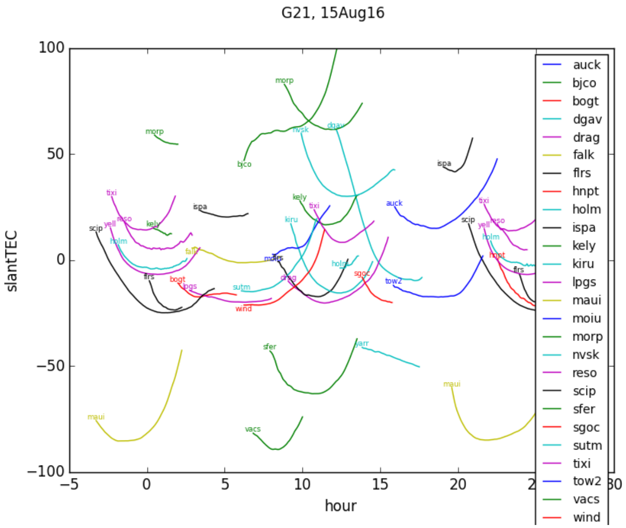
$$\rho(r,t) = \rho_{bgnd}(r,t)\, e^{\phi(r,t)}$$

$$\rho(r,t) = \sum_{\mu} a_{\mu} B_{\mu}(r,t)$$

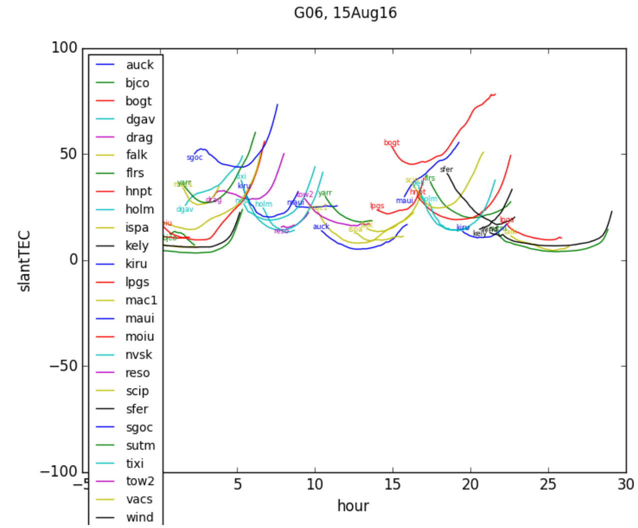
- **Tuning of finite-element weights can be driven by a Bayesian inference (MAP) process**
  - Weights sensor data and background trends according to level of confidence

- **“EDAM2” model assimilates GPS TEC into IRI-2016 background**
  - Tetrahedral grid techniques very similar to pure GPS tomography

Automatic estimation of GPS biases

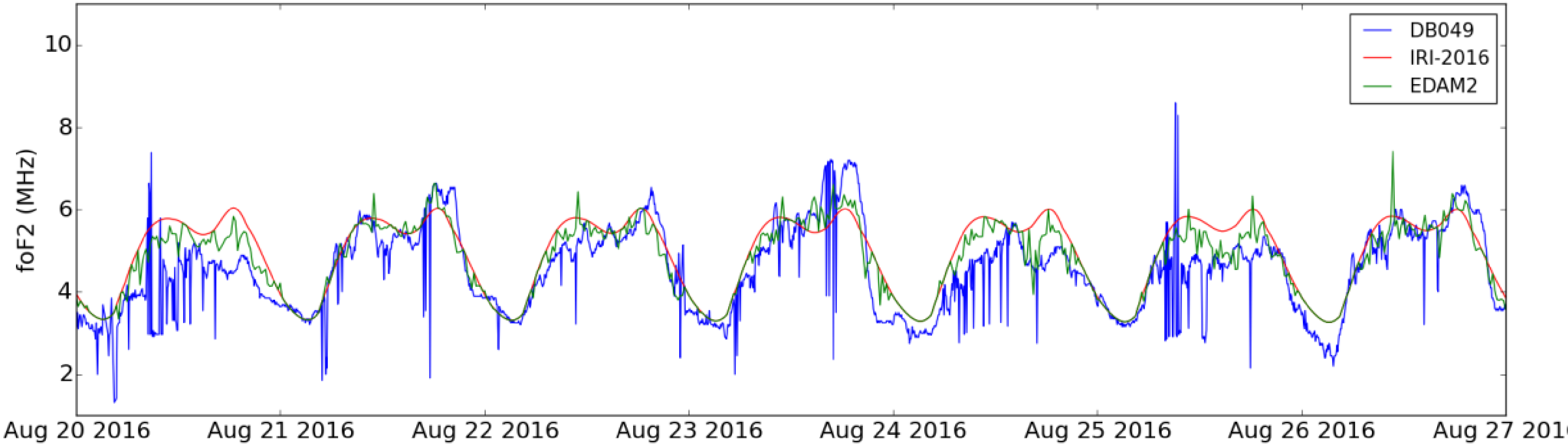


GPS bias parameters from EDAM2 successfully suppress unphysical TEC measurements

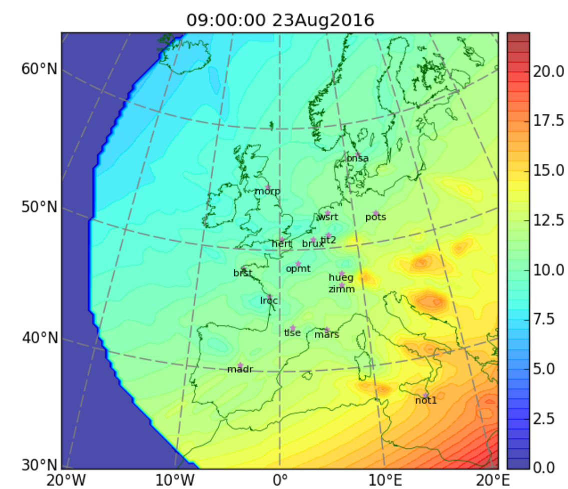


- **Inter-frequency time-delays need to be calibrated before real GPS TEC values can be used for tomography**
  - Using phase-rate avoids this, but makes for a more complicated tomographic model
- **GPS bias parameters can be incorporated into tomographic or assimilative model**
- **Suppression of negative TEC values is a basic sanity check of the tomographic fit**

Recovery of foF2 trends



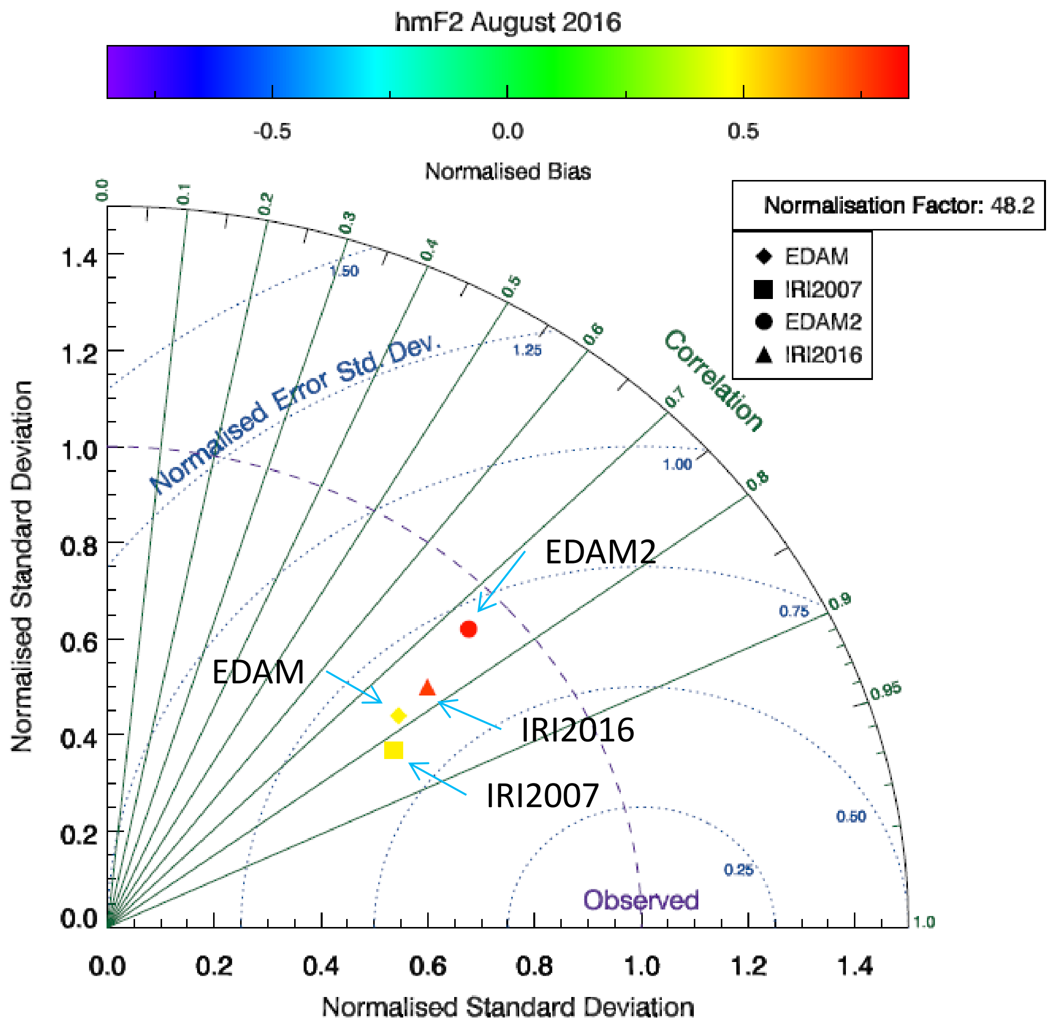
- **Test scenario assimilates 19 GPS stations across Europe, compared against 'DB049' ionosonde in Belgium**
  - DB049 is **not** used in the assimilation



- **Vertical profiles above DB049 provide direct comparison between "truth" and assimilative model**
  - Altitude (hmF2) and level (foF2) of maximum density provide convenient measure of success

Fidelity of hmF2

- Time-series of foF2 shows that estimation of bottom-side features can be improved using GPS
- Correlation of hmF2 with “truth” from ionosonde has been computed for entirety of August 2016
- EDAM2 model shows closer match of true variance than IRI or earlier EDAM model



Summary

- **Estimating 3D electron-density profiles is challenging given the sparsity of GPS and ionosonde data**
- **Rigorous treatment of interpolation is important to best use of sparse sensor data in tomographic fitting**
  - Popular piecewise-constant interpolation introduces artefacts and/or increases computational cost
- **Finite-element techniques provide an effective means of performing 3D tomography**
- **Pure tomographic fits to GPS data are able to recover large-scale ionospheric features**
  - Indicative of physical limits, independent of historical data or empirical profiles
- **Finite-element techniques can be used effectively in combination with background models (e.g. IRI) to allow statistical inference of 3D electron –density profile**

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