Properties of ULF Waves in Ionospheric Plasma

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ABSTRACT

The properties of low-frequency electromagnetic waves in the multicomponent ionospheric plasma in the 1-30 Hz band basing on the magneto ionic theory were examined. The permittivity tensor was calculated at altitudes from 80 to 1000 km. The results of calculation of the refractive indices of two normal waves (ordinary and extraordinary), the polarization ellipses and the peculiarities of the directivity of the group velocity vector with respect to the Earth magnetic field were presented. The validity of the application of the magneto ionic theory in the ionosphere for ULF waves is proved in the appendix.

1. INTRODUCTION

The propagation of ULF waves in the ionospheric plasma usually is treated in terms of the Alfven and fast magneto sonic waves in frame of magneto hydro dynamics since the first publication of Greifinger and Greifinger (1968). This approximation is used up today when one investigate the properties of the ionospheric MHD waveguide or the ionospheric Alfven resonator – IAR (see f. e. Polyakov and Rapoport (1981), Lysak, at all, (2013), Eliasson, at all, (2012)). In this paper the properties of ULF waves in ionospheric plasma are carefully investigated basing on the magneto ionic theory using the international models like IRI-2016 – for Ionosphere, MSIS-E-90 – for atmosphere and DGRF/IGRF – for geomagnetic field. The components of permittivity tensor, collisional frequencies, refractive index, polarization, the peculiarities of group velocity and wave normal surfaces for two ULF modes were calculated. The presented results demonstrate a great difference from one obtained in the magnetic hydrodynamics approach.

2. BASIC EQUATIONS

The permittivity tensor in the ULF band in the coordinate system with Z axis directed along magnetic field ***H*** can be presented by formulas (1) as it shown in the appendix:

$$\varepsilon_{xx} = \varepsilon_{yy} = 1 - \sum_{k=1}^{n} \left[\frac{\omega_{0k}^2}{2\omega} \left(\frac{1}{(\omega - \omega_{Hi}) - i\nu_{km}} + \frac{1}{(\omega + \omega_{Hk}) - i\nu_{km}} \right) \right]$$

$$\varepsilon_{xy} = -\varepsilon_{yx} = -i \sum_{k=1}^{n} \left[\frac{\omega_{0k}^2}{2\omega} \left(\frac{1}{(\omega + \omega_{Hk}) - i\nu_{km}} - \frac{1}{(\omega - \omega_{Hk}) - i\nu_{km}} \right) \right] \qquad (1)$$

$$\varepsilon_{zz} = 1 - \sum_{k=1}^{n} \left[\frac{\omega_{0k}^2}{\omega^2 - i\omega\nu_{km}} \right]$$

Hear: k – the kind of a charged particle (electrons and ions), $\omega = 2\pi * f$ – wave frequency,

$\omega_{0k} = \left(4\pi e^2 N_k(z)/m_k\right)^{1/2}$ – Longmire frequency, m_k – mass of particle,
$\omega_{Hk} = \pm eH(z)/m_k c$ gyrofrequency,(-) – electrons, (+) – ions,
ν_{km} – collision frequency of k particle with molecules and ions (see appendix)
$\underline{c}$ – light velocity, e – charge of electron, $H(z) = H_0 *(z/R_0)^3$ – Earth magnetic field,
R_0 = 6346 km –Earth radius, H_0 – field value on the Earth surface.

The refractive indexes of normal waves in ionospheric plasma at an arbitrary angle α between magnetic field and wave vector can be obtained by solving the equation (2)

$$\begin{aligned} &(n - i\kappa)^2_{1,2} = \frac{L \pm \sqrt{R}}{D}; \\ &L = (\varepsilon_{xx}^2 + \varepsilon_{xy}^2)\sin^2\alpha + \varepsilon_{xx}\varepsilon_{zz}(1 + \cos^2\alpha); \\ &D = 2(\varepsilon_{xx}\sin^2\alpha + \varepsilon_{zz}\cos^2\alpha); \\ &R = L^2 - 2D(\varepsilon_{xx}^2 + \varepsilon_{xy}^2)\varepsilon_{zz} \end{aligned} \tag{2}$$

The equation (2) is substantially simplified in the particular case of an electromagnetic wave propagating along magnetic field and takes the form (3), where 1 & 2 correspond to two modes of electromagnetic waves in the ionosphere.

$$(n - i\kappa)_{1,2} = \sqrt{\varepsilon_{xx}^2 \pm \varepsilon_{xy}^2} \tag{3}$$

It is also easy to show that in the lower ionosphere at E-region altitudes, where the ions can be considered as not magnetized, the real part of the refractive index of low-frequency waves coincides with the index for the whistler mode (the waves themselves are circularly polarized) and is given for vertical propagated wave by formula:

$$n = \frac{\omega_{0e}^2}{\omega_{H_{NO}} \cdot \omega \cdot Cos\alpha} \tag{4}$$

At altitudes above 150 km, the refractive index of one of the normal waves is close to the value of the refractive index for the FMS wave and the second for the Alfven wave, corrected by the proximity of the wave frequency to the ion gyrofrequency, and can be represented in the form:

$$n_1 = \frac{\omega_{0i}}{\omega_{Hi}}\left(1 + \frac{\omega^2}{\omega_{Hi}^2}\right); \quad n_2 = n_1 \cdot Cos\alpha \tag{5}$$

Where i – dominated ion at given altitude, $\omega < \omega_{Hi}$,.

In the limit ($\omega \to 0$), the first of the normal waves becomes a fast magnetosonic wave and the second one becomes Alfven wave. It is convenient to use (4) and (5) for estimations, but we have used formulas (1-2) for further modeling.

RESULTS OF CALCULATIONS

The parameters of upper atmosphere media were picked up from the mentioned above models. The mid latitude (Ψ=56^{O} & Λ=46^{O}) was chosen for definiteness. But the properties of ULF waves are weakly dependent on the latitude. The typical particles composition and electron collisional frequencies profiles were presented on the Fig.1.

Refractive indexes of normal ULF wave's modes calculated for ionospheric conditions from Fig.1 are shown on the Fig. 2. The main feature of these indexes is the dependence on frequency and also the strong attenuation of one of the mode in the E-layer. The peculiarities of the polarization properties of normal waves are presented on the Fig. 3.

It were calculated also the shapes of surfaces of the wave vectors and dependence of the angle α between magnetic field direction and the group velocity vector and angle θ between magnetic field and wave vector. The results of calculations are shown on the Fig. 4.

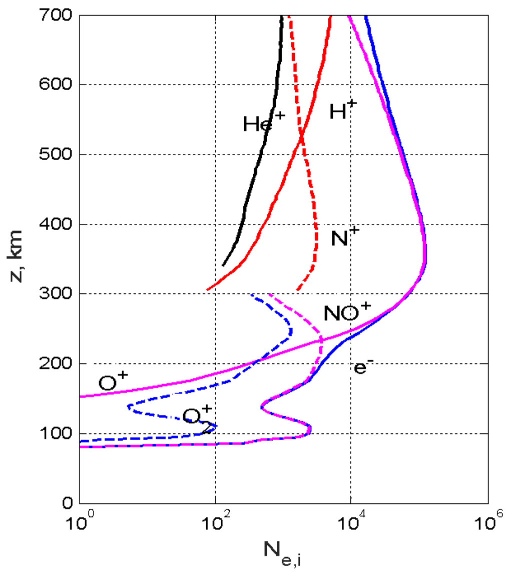

Fig. 1. Profiles of particles composition (left) & collisional frequencies (right)

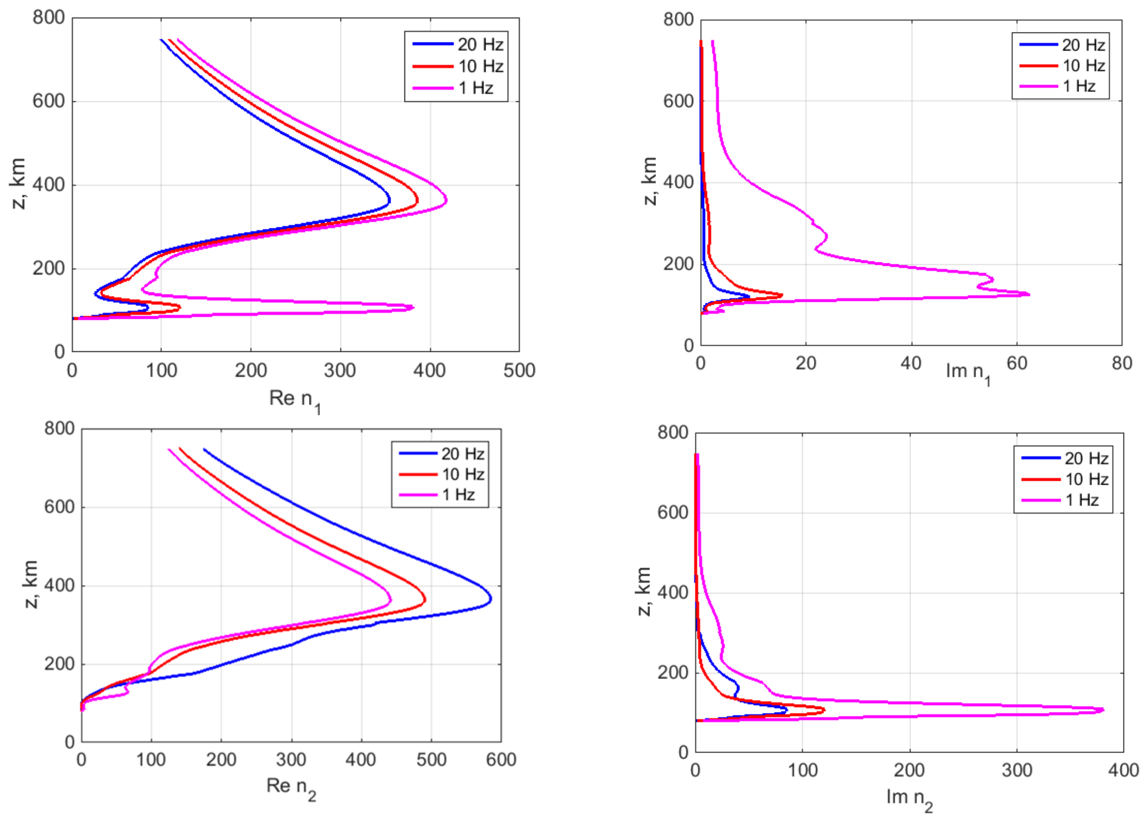

Fig. 2. Profiles of real part of normal waves (left) & imaginary part (right) of refractive indexes

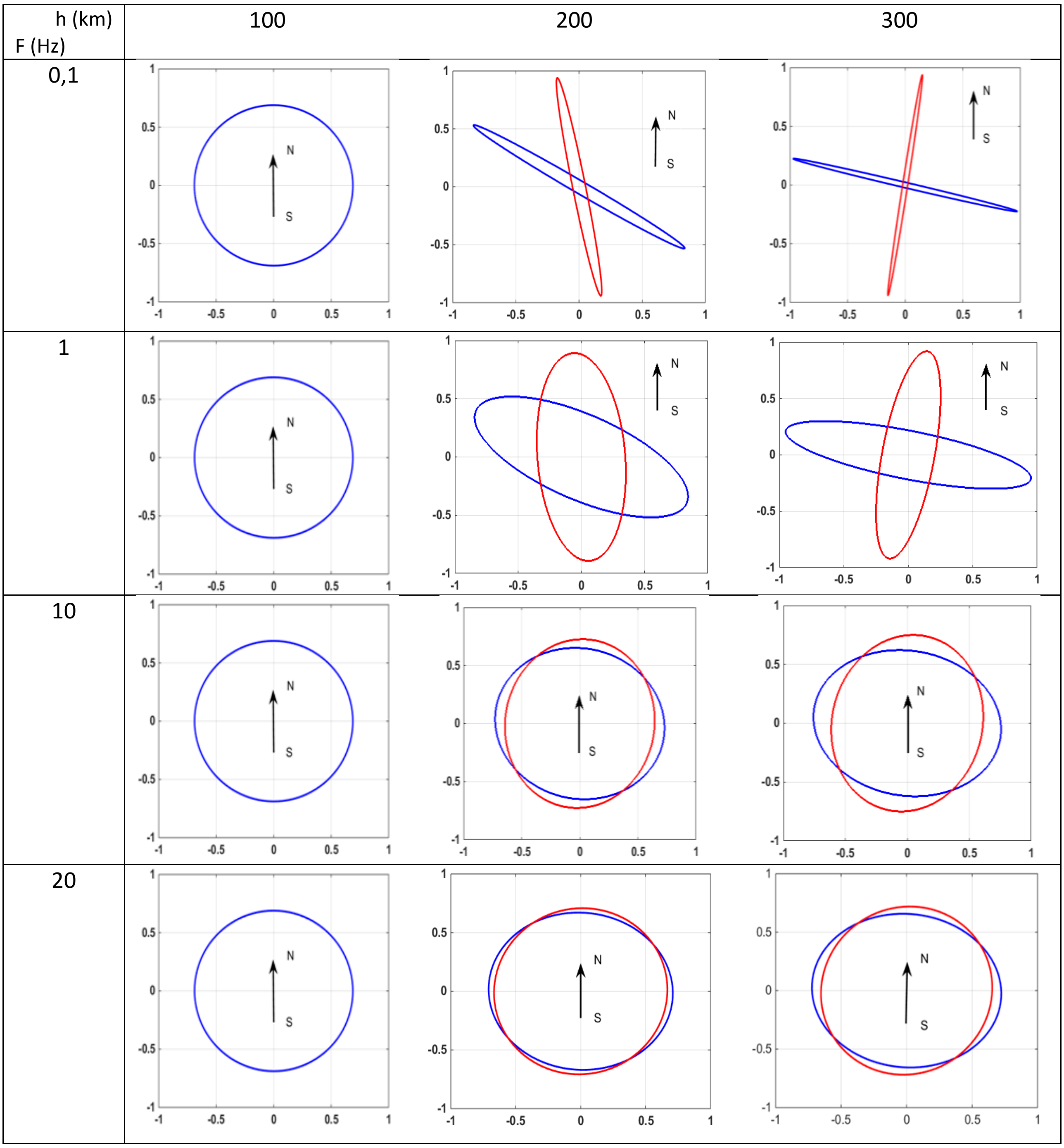

Fig. 3. Polarization`s properties of the ULF waves (blue – “FMS” wave, red – “Alfven” wave)

CONCLUSIONS

The results of calculation of the properties of ULF waves in ionospheric plasma are shown:

- The refractive indices of two normal waves (ordinary and extraordinary) are highly dependent on the frequency.
- The polarization of the two waves is elliptical in the whole range of investigated frequencies.

- The dependence of the group velocity of the “Alfven wave" on the angle between the wave vector and the Earth magnetic field is also differs from the MHD approximation.
- The refractive indexes and the polarization of normal waves tend to the values obtained in the magneto hydrodynamic approximation only at frequencies much lower than 1 Hz. Therefore, it is possible only conventionally to name one of them the Shear Alfven (SA) wave, and the second fast magneto sonic (FMS) wave. The "Alfven" wave in the lower ionosphere becomes strongly damped and the refractive index of FMS wave takes the form like for whistler mode and it is weakly damped in the lower ionosphere.
- The vector of the group velocity of “SA” waves in upper ionosphere is not directed along the magnetic field, but it is inside a cone within ± (20-25) degrees, depending on the frequency.
- The group velocity vector of the second wave corresponding to the FMS is practically independent of the angle with the magnetic field, as in the case of MHD approximatio

The account of the true characteristics of normal waves in the ionosphere is important for obtaining the correct results on propagation of the ULF waves in the presence of sharp boundaries on which the linear transformation of the normal modes takes place. For example, one have to keep it in mind when solving the exit problem of ULF waves from ionospheric and magnetospheric sources to the Earth's surface through the complicate structure of the ionospheric Alfven resonator and when modeling it properties (see f. e. Ermakova at all, (2013) and other works of this group of authors).

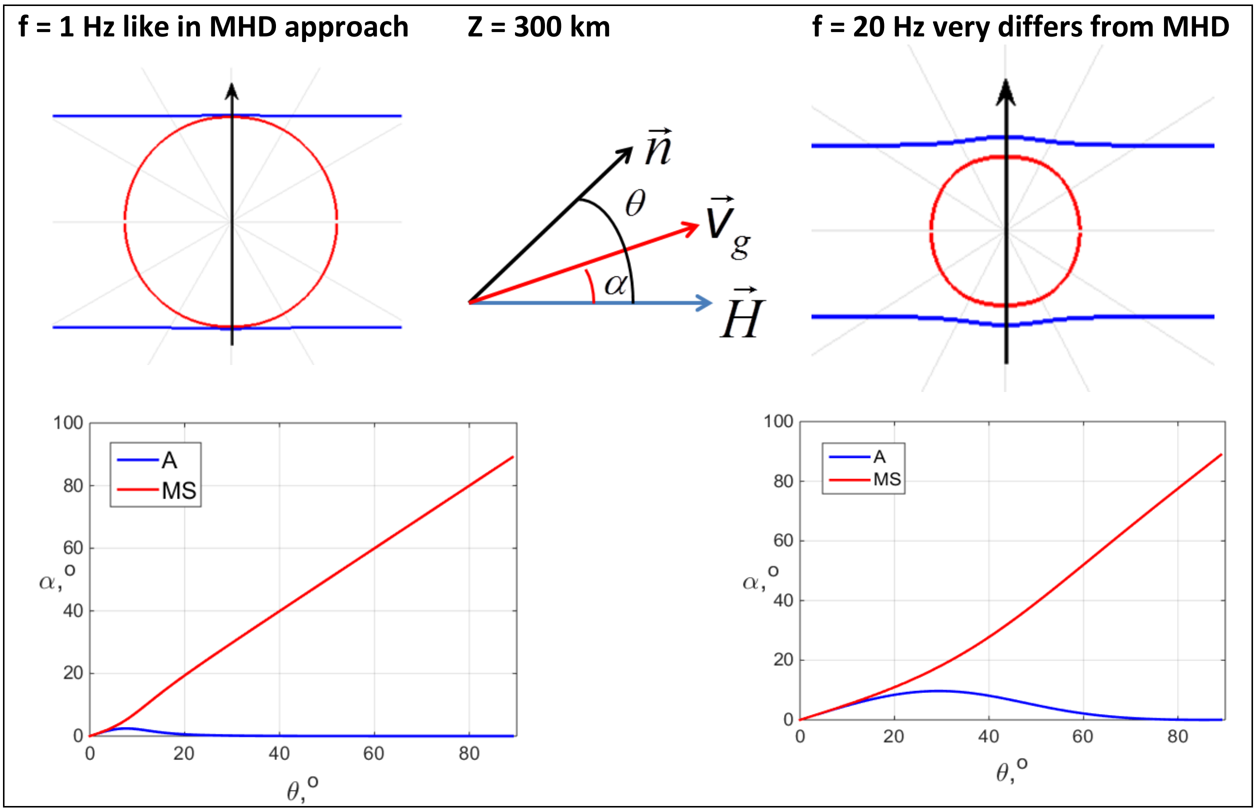

Fig. 4. Shapes of surfaces of wave vektors (upper panel) and dependence of the angle α between magnetic field direction and group velocity vector and angle θ between magnetic field and wave ve.Group velocity vector of ULF wave (lower panel).

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APPENDIX

The usage of the magneto-ionic theory to calculate the components of the permeability tensor is not obvious. The initial system of equations (10.34) from the Ginzburg book was taken for calculations:

$$-i\omega\vec{\mathrm{v}}_{\mathrm{e}} = \frac{e}{m}\vec{E} + \frac{e}{mc}\left[\vec{\mathrm{v}}_{\mathrm{e}}\vec{H}\right] + \nu_{ei}(\vec{\mathrm{v}}_{\mathrm{i}}\text{-}\vec{\mathrm{v}}_{\mathrm{e}}) + \nu_{em}(\vec{\mathrm{v}}_{\mathrm{m}}\text{-}\vec{\mathrm{v}}_{\mathrm{e}}) \tag{A.1}$$

$$-i\omega\vec{\mathrm{v}}_{\mathrm{i}} = -\frac{e}{M}\vec{E} - \frac{e}{Mc}\left[\vec{\mathrm{v}}_{\mathrm{i}}\vec{H}\right] + \frac{m}{M}\nu_{ei}(\vec{\mathrm{v}}_{\mathrm{e}}\text{-}\vec{\mathrm{v}}_{\mathrm{i}}) + \nu_{im}(\vec{\mathrm{v}}_{\mathrm{m}}\text{-}\vec{\mathrm{v}}_{\mathrm{i}}) \tag{A.2}$$

$$-i\omega\vec{\mathrm{v}}_{\mathrm{m}} = -\frac{mN}{MN_m}\nu_{em}(\vec{\mathrm{v}}_{\mathrm{m}}\text{-}\vec{\mathrm{v}}_{\mathrm{e}}) + \frac{N}{N_m}\nu_{im}(\vec{\mathrm{v}}_{\mathrm{m}}\text{-}\vec{\mathrm{v}}_{\mathrm{i}}) \tag{A.3}$$

Here $\mathrm{v}_{(k)}$ – velocities of elections, ions and molecules, m & M – *masses* of elections and ions, H – magmatic field.
One can obtain from this system formula for molecules velocity:

$$\vec{\mathrm{v}}_{\mathrm{m}} = \left(\frac{mN}{MN_m}\nu_{em}(\vec{\mathrm{v}}_{\mathrm{e}}\text{-}\vec{\mathrm{v}}_{\mathrm{i}}) + \frac{N}{N_m}\nu_{im}(\vec{\mathrm{v}}_{\mathrm{m}}\text{-}\vec{\mathrm{v}}_{\mathrm{i}}) \right)\cdot\left(-i\omega + \nu_{im} N/N_m\right)^{-1} \tag{A.4}$$

The condition $\omega >> \nu_{im} N/N_m$ performed in the lower ionosphere, with a large margin $(\approx 10^{-5})$

$$\omega >> \nu_{im} N/N_m \tag{A.5}$$

That allows to exclude from (A1, A2) terms containing the velocity of the molecules что позволяет исключить из (1, 2) члены, содержащие скорость молекул.

Taking into account the last inequality and neglecting terms of order $m/M, \sqrt{m/M}$, also taking into account inequality $\nu_{im} << \nu_{em}\sqrt{m/M_i}$, the equations (A1, A2) take the form which coincides with that obtained in the magneto ionic theory and look like:

$$(-i\omega + \nu_{em} + \nu_{ei})\vec{\mathrm{v}}_{\mathrm{e}} = \frac{e}{m}\vec{E} + \frac{e}{mc}\left[\vec{\mathrm{v}}_{\mathrm{e}}\vec{H}\right] \tag{A.6}$$

$$(-i\omega + \nu_{km})\vec{\mathrm{v}}_{\mathrm{k}} = -\frac{e}{m_k}\vec{E} - \frac{e}{m_k c}\left[\vec{\mathrm{v}}_{\mathrm{k}}\vec{H}\right] \tag{A.7}$$

Now it is easy to obtain the from (A6, A7) the components of the permeability tensor (1)

The semiempirical collision formulas were taken from the book (Fatkullin at all. Empirical models of the mid-latitude ionosphere, Moscow. Nauka, 1981, 256p).

For collisions of electrons with molecules:

$$\begin{aligned} &\nu(e,O_2) = 1.82\cdot10^{-10}N(O_2)(1+3.6\cdot10^{-2}\sqrt{T_e})\sqrt{T_e}; \\ &\nu(e,N_2) = 2.33\cdot10^{-11}N(N_2)(1-1.21\cdot10^{-4}T_e)T_e; \\ &\nu(e,O) = 4.5\cdot10^{-10}N(O)\sqrt{T_e} \\ &\nu_{em} = \nu(e,O_2)+\nu(e,N_2)+\nu(e,O) \end{aligned} \tag{A.9}$$

For collisions of electrons with ions:

$$\nu_{ei}=54*N_e./(T_e^{(3/2)}) \quad \text{(A.10);}$$

For ions O_2^+ with molecules:

$$\begin{aligned} &\nu(O_2^+,O_2) = 1.17\cdot10^{-9}\left(\frac{T_i+T_n}{2000}\right)^{0.28}; \\ &\nu(O_2^+,O,N_2) = (0.75\cdot N(O)+0.89\cdot N(N_2))\cdot10^{-9} \\ &\nu_{O_2^+m} = \nu(O_2^+,O_2)+\nu(O_2^+,O,N_2) \end{aligned} \tag{A.11}$$

For ions NO^+ with molecules::

$$\nu_{NO^+m} = 10^{-9}(0,83N(O_2)+0.76N(NO)+0.76N(N_2)) \tag{A.12}$$

For ions O^+ with molecules:

$$\nu_{O^+m} = 10^{-9}(N(NO)+1.08N(N_2)) \tag{A.13}$$